

MARITIME TRANSPORTATION RESEARCH AND EDUCATION  
CENTER  
TIER 1 UNIVERSITY TRANSPORTATION CENTER  
U.S. DEPARTMENT OF TRANSPORTATION



## Optimal Dredge Fleet Scheduling within Environmental Work Windows

MarTREC 5002  
08/01/14 – 08/15/16

**Principal Investigators**  
Chase Rainwater, Ph.D.  
cer@uark.edu

Heather L. Nachtmann, Ph.D.  
hln@uark.edu

**Graduate Researcher**  
Fereydoun Adbesh, Ph.D. Candidate

Department of Industrial Engineering - University of Arkansas  
Fayetteville, AR 72701

**September 15, 2016**

**FINAL RESEARCH REPORT**

Prepared for:  
Maritime Transportation Research and Education Center

University of Arkansas  
4190 Bell Engineering Center  
Fayetteville, AR 72701  
479-575-6021

## **ACKNOWLEDGEMENT**

This material is based upon work supported by the U.S. Department of Transportation under Grant Award Number DTRT13-G-UTC50. The work was conducted through the Maritime Transportation Research and Education Center at the University of Arkansas.

## **DISCLAIMER**

The contents of this report reflect the views of the authors, who are responsible for the facts and the accuracy of the information presented herein. This document is disseminated under the sponsorship of the U.S. Department of Transportation University Transportation Centers Program, in the interest of information exchange. The U.S. Government assumes no liability for the contents or use thereof.

## Abstract

The U.S. Army Corps of Engineers (USACE) annually spends more than 100 million dollars on dredging hundreds of navigation projects on more than 12,000 miles of inland and intra-coastal waterways. Building on previous work with USACE, this project expands logic-based mathematical programming solution approaches to address more realistic dredge scheduling challenges faced by the USACE.

In previous work, both mixed-integer and constraint programming formulations were developed to allocate dredge resources to projects system-wide while adhering to various limitations such as so-called environmental restrictions that define when dredging cannot take place due to migration patterns of different species (e.g. turtles, birds, fish, and other wildlife). In addition, dredge equipment resource availability and varying equipment productivity rates that impact project completion times were considered.

In this study we extend the previously developed constraint programming model, adding flexibility to address more of USACE's needs. The extended model allows for partial dredging during restricted periods, variable jobs sizes, multiple dredges working on a job, multiple dredging equipment trips to the same job, dredge maintenance and varying operation rates/costs. The result of our research is a more applicable decision tool that can be used by USACE to determine the appropriate dredge fleet and the optimal operations associated with that fleet for a given set of jobs and their characteristics.

# Contents

Abstract	i
<b>1 Project Description</b>	<b>1</b>
1.1 Introduction . . . . .	1
1.2 Research Motivation and Prior Work . . . . .	1
<b>2 Methodological Approach</b>	<b>3</b>
2.1 Mixed Integer Programming (MIP) Formulation . . . . .	3
2.2 Base Constraint Programming (CP) Formulation . . . . .	6
2.3 Why CP? . . . . .	9
<b>3 Project Findings</b>	<b>10</b>
3.1 Problem Instances Description . . . . .	10
3.2 Extension and Improvement . . . . .	12
3.2.1 Allowing Partial Dredging During Restricted Periods . . . . .	12
3.2.2 Variable Job Sizes . . . . .	22
3.2.3 Multiple Trips to the Same Job . . . . .	24
3.2.4 Multiple Dredges on the Same Job . . . . .	26
3.2.5 Different Operation Rates, Unit Cost of Dredging and Budget . . . . .	30
3.2.6 Simulating Downtime for Dredges . . . . .	31
3.2.7 Mob/Demob Cost . . . . .	32
3.2.8 Dredge Capabilities by Job . . . . .	33
<b>4 Impacts of Implementation</b>	<b>33</b>
4.1 The Modified Comprehensive Model . . . . .	33
<b>5 Recommendations and Conclusions</b>	<b>41</b>

## List of Figures

1	Binary Variable vs. Interval Variable . . . . .	10
2	Graphical Depiction of 116 Dredge Jobs Locations . . . . .	11
3	Interval Variables with the Same Sizes and Different Lengths . . . . .	14
4	All RPs Relaxation Impacts on Amount of Dredging . . . . .	21
5	All RPs Relaxation Impacts on Time . . . . .	22
6	Impact of Allowing Variable Job Sizes . . . . .	24

## List of Tables

1	Number of Variables and Constraints in MIP and CP Formulation . . . . .	9
2	Summary of Restricted Periods . . . . .	12
3	Individual Impacts of RPs Relaxation . . . . .	15
4	Individual Impacts of RPs Relaxation with Initial Solution . . . . .	16
5	All RPs Relaxation Impacts . . . . .	20
6	All RPs Relaxation Impacts with Initial Solution . . . . .	21
7	Variable Jobs Size . . . . .	23
8	Multiple Trips to the Same Jobs . . . . .	26
9	Multiple Dredges on the Same Jobs with Different Rates . . . . .	29
10	Different Operation Rates and Cost of Dredging for each Job . . . . .	31
11	Individual Impacts of RPs Relaxation in MCP-DS . . . . .	36
12	All RPs Relaxation Impacts in MCP-DS . . . . .	40
A1	116 Project Properties . . . . .	44
A2	Production Rates of Dredge Vessels . . . . .	46

# 1 Project Description

## 1.1 Introduction

The U.S. Army Corps of Engineers (USACE) conducts maintenance dredging at hundreds of navigation projects each year to provide safe, reliable, and cost efficient waterborne transportation systems with minimal impact on the environment. Most of these dredging jobs are for movement of commerce, national security needs, and recreation. These primary navigation responsibilities include planning and constructing new navigation channels, locks and dams. In addition, the corps is charged with planning and executing dredging operations necessary to maintain navigable channel depths at U.S. harbors and on inland waterways. These dredging efforts include nearly 12,000 miles of inland and intracoastal waterway navigable channels, including 192 commercial lock and dam sites in 41 states. The Corps dredges over 250 million cubic yards of material each year at an average annual cost of over \$1.3 billion to keep the nation's waterways navigable [10].

To protect against harm to local environmental species, dredging jobs must adhere to environmental restrictions which place limits on how and when dredging may occur. For example, west coast Ventura Harbor may not be dredged from April to August to avoid harming California Grunion living in the area.

Allocating dredging vessels (whether government or private industry) to navigation projects is typically made at the corps district-level by assigning the projects with the intent on maximizing the amount of dredging completed over a calendar year. The U.S is divided into 38 Corps Districts, generally along watershed and state boundaries, and the resulting dredge-selection process is typically decentralized. Within each district, decision-makers are challenged with the task of determining a desirable resource allocation portfolio that adheres to budgetary restrictions, protects the environment and transports/operates equipment according to a strategic schedule. The work that follows in this report offers an enhanced quantitative tool to meet this challenge.

## 1.2 Research Motivation and Prior Work

Nachtmann et al. [8] developed one of the first optimization tools to improve dredge scheduling decision-making process. The approach utilized a customized constraint/integer programming approach that was shown to provide quality solutions to problems with 100+ jobs in reasonable time. In their problem, the following problem characteristics were considered:

1. **Dredge transportation:** Each dredge must move between jobs and the resource is

not available for operation during these times.

2. **Dredging budget:** The dredging plan (transportation, operation) chosen must adhere to a single budgetary restriction.
3. **Environmental windows:** Dredging is not allowed during specified time intervals.

While these characteristics form the fundamental requirements for dredging, there are significant opportunities for expansion and improvement of the developed optimization approach. The USACE partners identified numerous realistic considerations that need to be considered. Specifically, the prior tool [8] suffered from the following limitations:

1. **Partial dredging during environmental windows:** Environmental windows prevent any dredging from occurring over a specified horizon; i.e. partial dredging is not allowed during the environmental windows. This restriction limits the capability of available resources to conduct their dredging jobs dramatically. For example, according to a real problem instance in Section 3.1, 6 jobs out of 116 jobs cannot be dredged at all according to their environmental windows restrictions.
2. **Variable job sizes:** Jobs must be fully completed by a dredge vessel before moving on to other jobs. This means partial jobs are not allowed and the size of jobs cannot be varied. In practice, there is often a “minimum requirement” that really should attempt to be met as well as a “target requirement” that would be ideal to get if time/money allows.
3. **Multiple dredges on the same job:** Jobs must be satisfied by a single dredge vessel. In some situation, however, jobs need to be done by different types of dredge vessels.
4. **Multiple trips to the same jobs:** Jobs must be satisfied by a single dredge vessel trip; i.e. a dredge vessel cannot begin operating on a job if it cannot be finished before the beginning of its environmental window(s) or the end of the overall time horizon. This restriction prevents us from dredging a job in multiple trips.
5. **Different operation rates for each job:** The operation rates of dredge vessels are constant in all dredging jobs. The cubic yards of dredging per hour for each dredge vessel can vary greatly from one job to the next due to wave conditions, weather, sediment types, and etc.

6. **Different unit costs of dredging for each job:** The dredging cost per cubic yard is constant in all jobs. Practically, the cost rate of dredging is different for each job-vessel combination due to specifications of each job (e.g. the width and depth), type of sediments, and fuel cost.
7. **Simulating downtime for dredges:** Dredge vessels typically have fixed amounts of time in the yards for repairs and consequently are not available.
8. **Dredge capability on jobs:** Some dredges cannot perform some of the jobs. Some dredges cannot physically perform the dredging job in some locations. For instance, some of dredges in the fleet are too big to maneuver between some small ports.

In Section 2.2, the previous constraint/integer programming approach and results studied by Nachtmann et al. [8] is presented which is the foundation of our models and base model to compare the results. In Section 3.2, we investigate the abilities of constraint programming to allow for multiple-resource job processing and proposing more sophisticated ways to model environmental constraints beyond the “fully restrictive” enforcement currently employed. In Section 5 our future plan to study on the remaining weaknesses is presented.

## 2 Methodological Approach

### 2.1 Mixed Integer Programming (MIP) Formulation

The Mixed Integer Programming (MIP) formulation of the dredge fleet scheduling problem with environmental work windows introduced in Nachtmann et al. [8] is presented in this section. This Dredge Scheduling (DS) model formulation assigns available dredge vessels to unsatisfied dredging jobs to maximize the total cubic yards of dredging over a finite planning horizon. All dredging jobs can only happen in the environmental work windows of the jobs and dredging is prohibited in the restricted periods of the jobs (RPs). Thus, environmental windows and RPs are complementary definitions.

**Sets:**

- $d \in D$ , set of dredging equipment resources available in each time period,
- $t \in T$ , set of consecutive time periods comprising the planning horizon,
- $j \in J$ , set of dredge jobs that need to be completed over the planning horizon, and
- $w \in W_j$ , set of RPs applicable to dredging job  $j$ .

**Parameters:**

- $b_w$ , the beginning of RP  $w$ ,  $w \in W_j, j \in J$ ,
- $e_w$ , the end of RP  $w$ ,  $w \in W_j, j \in J$ ,
- $r_d$ , the operation rate (cubic yards/day) of dredge equipment  $d \in D$
- $q_j$ , the dredging amount of job  $j \in J$  (in cubic yards),
- $t_{jd} = \lceil q_j/r_d \rceil$ , the time (days) that it takes for dredge equipment piece  $d \in D$  to complete job  $j \in J$ ,
- $t_{jj'}$ , the time (days) that it takes to move a dredging equipment piece  $d \in D$  from job site  $j \in J$  to job site  $j' \in J, j \neq j'$ ,
- $c_j$ , the cost for completing job  $j \in J$ , and
- $B$ , the available budget for the planning horizon.

**Decision Variables:**

- $y_{dj}$ , binary variable equals 1 if dredging vessel  $d$  is used to complete job  $j$ , and
- $z_{djt}$ , binary variable with value 1 if dredging vessel  $d$  begins work on job  $j$  in period  $t$ .

$$\max \sum_{j \in J} \sum_{d \in D} q_j y_{dj}$$

subject to

$$\sum_{d \in D} y_{dj} \leq 1 \quad j \in J \quad (1)$$

$$\sum_{j \in J} \sum_{d \in D} c_j y_{dj} \leq B \quad (2)$$

$$\sum_{t \in T} z_{dtj} = y_{dj} \quad j \in J, d \in D \quad (3)$$

$$\sum_{t' = t}^{\min\{T, t + t_{jd} + t_{j'j'}\}} z_{dj't'} \leq 1 - z_{dj't} \quad j, j' \in J, j \neq j', d \in D, t \in T \quad (4)$$

$$\sum_{d \in D} \sum_{t = \max\{1, b_w - t_{jd}\}}^{e_w} z_{dj't} = 0 \quad w \in W_j, j \in J \quad (5)$$

$$(t + t_{jd}) z_{dj't} \leq |T| \quad j \in J, d \in D, t \in T \quad (6)$$

$$y_{dj} \geq 0 \quad d \in D, j \in J \quad (7)$$

$$z_{dj't} \in \{0, 1\} \quad d \in D, j \in J, t \in T \quad (8)$$

The objective of the model is to maximize the total dredging amount in cubic yards over the planning horizon. Constraints (1) ensure that job  $j$  is satisfied by at most one dredging vessel  $d$ , whereas Constraint (2) imposes that the total cost of dredge assignments cannot exceed the total budget ( $B$ ). Constraints (3) require that if job  $j$  is satisfied by vessel  $d$ , exactly one start day for job  $j$  must be specified for assignment  $d$ . Constraints (4) specify that if job  $j$  is started in period  $t$ , by vessel  $d$ , then vessel  $d$  cannot begin another job,  $j'$ , until  $t_{jj'} + t_{jd}$  periods (days) have passed (where  $t_{jj'}$  is the time to complete job  $j$  on dredge vessel  $d$  and  $t_{jd}$  is the time to travel to job  $j'$  from job  $j$ ). Constraints (5) prevent a job from starting or ending during its restricted period(s). Constraints (6) ensure that if a job is to be processed, the completion time should be before the end of the planning horizon. Finally, Constraints (7)-(8) specify the appropriate domain of each variable in the model.

Similar to other applications of many integer programs, as the number of decision variables and constraints increases, finding an optimal solution become more challenging. A commercial optimization solver, ILOG CPLEX, on a Core 2 Duo 2.93 GHz, 16 GB RAM computer cannot even start solving a medium sized instance of this MIP model (10 dredge vessels and 32 jobs) due to lack of memory to load all required decision variables and con-

straints. To overcome this limitation, a constraint programming (CP) model formulation (CP-DS) is proposed by Nachtmann et al. [9]. Similar to other scheduling applications the CP-DS model produced high-quality feasible solutions within a reasonable amount of computational time. The descriptive formulation and details are discussed in Section 2.2.

## 2.2 Base Constraint Programming (CP) Formulation

Nachtmann et al. [9] introduced a constraint programming model for the dredge scheduling problem (CP-DS) to find high quality feasible solutions for real sized problems with over 100 jobs and 30 dredge vessels. This approach sought to overcome the inability of ILOG CPLEX solver to even load all the variables and constraints of the MIP formulation presented in the previous section.

In the CP-DS model, the time-dependent binary variables modeled through the use of *global constraints* and *interval variables*. An interval variable represents an interval of time during which an operation occurs [3]. More details on interval variables will be discussed in Section 3.2.1.

CP Optimizer, a constraint programming solver engine developed by ILOG, solves a model using constraint propagation and constructive search with search strategies [3]. Conveying information between constraints and variables is made possible by constraint propagation (filtering) iterative processes of global constraints. Each global constraint is associated with a propagation algorithm to remove the values of variables from their domains (van Hoes and Katriel [13], Hooker [6]). The propagation algorithm is executed after each variable change. Since constraints are related to each other through shared variables, whenever a change occurs on the domain of a shared variable due to the propagation algorithm of a constraint, the filtering algorithms of other constraints are also triggered to evaluate possible other reductions in the domains of all variables (Lombardi and Milano [7], Harjunkoski and Grossmann [5]). Branching on an individual variable takes place only after all possible reductions on domains are made.

Topaloglu and Ozkarahan [11], and van Hoes and Katriel [13] summarize the differences between ILP and CP. The most significant difference noted is the way that inference is performed at every branch node. In CP, inference at each node is done by constraint propagation. At each branching node, domains of *all* variables are reduced by the propagation algorithms of constraints. In CP, a node is pruned in the case of an empty domain of any variable obtained by the constraint propagation. Additional branching is needed when a variable's domain has more than one element or the bounds are outside of a specified range.

The CP-DS formulation of the dredge scheduling problem introduced by Nachtmann et

al. [9] is presented as follows. The following parameters and decision variables are used in developing the CP formulation.

**Sets:**

- $d \in D$ , set of dredging equipment resources available in each time period,
- $t \in T$ , set of consecutive time periods comprising the planning horizon,
- $j \in J$ , set of dredge jobs that need to be completed over the planning horizon, and
- $w \in W_j$ , set of RPs applicable to dredging job  $j$ .

**Parameters:**

- $b_w$ , the beginning of RP  $w$ ,  $w \in W_j, j \in J$ ,
- $e_w$ , the end of RP  $w$ ,  $w \in W_j, j \in J$ ,
- $r_d$ , the operation rate (cubic yards/day) of dredge equipment  $d \in D$
- $q_j$ , the dredging amount of job  $j \in J$  (in cubic yards),
- $t_{jd} = \lceil q_j/r_d \rceil$ , the time (days) that it takes for dredge equipment piece  $d \in D$  to complete job  $j \in J$ ,
- $t_{jj'}$ , the time (days) that it takes to move a dredging equipment piece  $d \in D$  from job site  $j \in J$  to job site  $j' \in J, j \neq j'$ ,
- $c_j$ , the cost for completing job  $j \in J$ , and
- $B$ , the available budget for the planning horizon.
- $I(j)$ , the *Intensity Function* [4] of job  $j \in J$ . That is  $I(j) = 0\%$ , if the job  $j$  is not allowed to be processed at time  $t$  such that  $b_w \leq t \leq e_w$ ,  $I(j) = 100\%$  otherwise.
- $TD[\text{Type}(j), \text{Type}(j')]$ , the *Transition Distance* between job  $j \in J$  and  $j' \in J$ . It is used to inform other global constraints that the travel time between job pairs  $j$  and  $j'$  should be at least  $t_{jj'}$ .

**Decision variables:**

- $y_{jd}$ , optional interval variable when job  $j \in J$  (with size  $q_j$ ) is assigned to dredge vessel  $d \in D$ ,

- $Y_j = \{y_{j1}, y_{j2}, \dots, y_{jD}\}$ , set of interval variables representing possible dredge equipment  $d \in D$  that can be assigned to job  $j \in J$ ,
- $Y_d = \{y_{1d}, y_{2d}, \dots, y_{Jd}\}$ , set of interval variables representing possible jobs  $j \in J$  that can be assigned to dredge vessel  $d \in D$  (the *interval sequence variable* for  $d$ ),
- $z_j$ , optional interval variable associated with job  $j \in J$ .

$$\max \sum_{j \in J} q_j z_j$$

subject to

$$\text{Alternative}(z_j, Y_j) \quad j \in J \quad (9)$$

$$\text{Cumulative}(z_j, c_j, B) \quad (10)$$

$$\text{Cumulative}(z_j, 1, |D|) \quad (11)$$

$$z_j.\text{StartMin} = 1 \quad j \in J \quad (12)$$

$$z_j.\text{EndMax} = |T| \quad j \in J \quad (13)$$

$$\text{ForbidExtend}(z_j, I(j)) \quad j \in J \quad (14)$$

$$\text{NoOverlap}(Y_d, TD[\text{Type}(j), \text{Type}(j')]) \quad d \in D \quad (15)$$

The objective function above seeks to maximize the total dredged amount in cubic yards. Constraints (9) ensure that each job can only be assigned to at most one dredge vessel by choosing exactly one possible assignment from all possible assignments of dredge vessels to job  $j$ . The *Alternative* global constraints enforce if an interval decision variable  $z_j$  is present in the solution then one and only one of the elements of  $Y_j$  array of interval variables must be presented in the solution.

Constraint (10) states that the total cost of dredging operations cannot exceed the total budget  $B$ . A CP *Cumulative* constraint models the resource usage over time and is computed using sub-functions such as *Step*, *Pulse*, *StepAtStart* and *StepAtEnd* [4]. In the programming of (CP-DS) formulation, *StepAtStart*( $z_j$ ) increases the total money spent on operations at the start of interval variable  $z_j$  by the amount  $c_j$ . Constraint (10) ensures the total cost does not exceed the available budget. Similarly, in Constraint (11), the *Cumulative* global constraint, in conjunction with the *Pulse*( $z_j$ ) function, is used to make sure that total number of occupied dredge vessels at any time cannot exceed the fleet size  $|D|$ . Constraints (12) and (13) specify the minimum start time and maximum end time of each job to the first and last

day of the planning horizon, respectfully. The ForbidExtend Constraint (14) prevents job  $j$  from being performed during its restricted period(s)  $I(j)$ . On the other hand, if interval variable  $z_j$  is presented in the solution, it cannot overlap with the time intervals where its intensity function is 0%. Finally, the *NoOverlap* Constraints (15) ensure that, if both jobs  $j$  and  $j'$  are operated by dredge vessel  $d$ , then a minimal time  $t_{jj'}$  must be maintained between the end of interval variable  $y_{jd}$  and the start of the interval variable of  $y_{j'd}$  and otherwise.

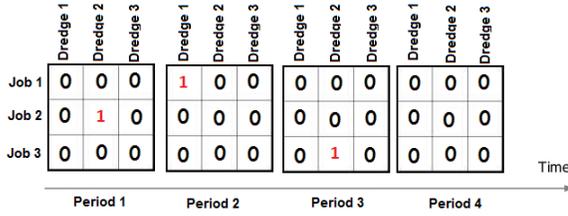
## 2.3 Why CP?

The main reason constraint programming was used in this work is the desire to solve large, more complex, problems. The base problem instance, which will be discussed thoroughly in Section 3.1, has 116 jobs with 30 dredges and a time horizon of 365 days (with associated restricted periods to each job). The dredge scheduling problem with time windows can be converted to the parallel machine scheduling problem with sequence dependent setup times and job availability intervals [2], which is known to be an NP-Complete problem. Table 1 shows the number of variables and constraints of the base model instance in both the mathematical formulation of MIP and CP.

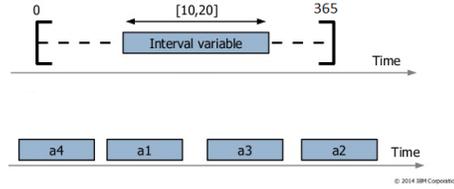
Table 1: Number of Variables and Constraints in MIP and CP Formulation

<b>Mathematical Formulation</b>	<b>Number of Variables</b>	<b>Number of Constraints</b>
MIP	1,273,680	1,741,276
CP	3,626	380

As we can see in Table 1, the number of variables and constraints in the MIP formulation is dramatically more than the CP formulation. This prevents CPLEX solver from loading the data into its memory. In contrast, CP solver can find a quality solution in a reasonable amount of time (60 seconds). The main reason for the difference between the number of variables and constraints is using binary variables in MIP formulation versus interval variables in CP. In the MIP formulation (Section 2.1) we use  $z_{djt}$  binary variables with value 1 if dredging vessel  $d$  begins work on job  $j$  in period  $t$ ,  $\forall d \in \{1, 2, \dots, 30\}$ ,  $j \in \{1, 2, \dots, 116\}$ , and  $t \in \{1, 2, \dots, 365\}$ . As a small example, consider a binary variable  $z_{djt}$  with 3 jobs, 3 dredges and 4 periods.



(a) Binary Variable  $z_{djt}$



(b) Interval Variable  $z_j$  (Source: 2014 IBM Corporation)

Figure 1: Binary Variable vs. Interval Variables

Figure 1 (a) shows a sample solution of the small instance problem using a MIP formulation. In this solution, dredge 2 starts working on job 2 in period 1 and then travels to job 3 and start working on it in period 3. Also, dredge 1 starts working on job 1 at period 2. As we can see in this small example, 36 (3 jobs  $\times$  3 dredges  $\times$  4 periods) binary variables exist in the MIP formulation and just 3 of them will become equal to 1 in the final solution. Thus, we are creating many variables that will simply take on the value of 0. This consumes the CPLEX solver’s memory and makes our scheduling combinatorial problem intractable. In contrast, in the CP formulation of the problem, only 3 interval variables associated with each job exist. A typical interval variable is shown in Figure 1 (b). Each interval variable carries a possible range, start and finish time value and size. This reduces the number of variables in comparison with MIP formulation to solve the problem. Moreover, interval variables can be set in any sequence for scheduling purposes.

### 3 Project Findings

#### 3.1 Problem Instances Description

In this section the data collection to establish problem instances and computational results of performing the CP-DS model is presented. The data was provided by the USACE Dredging Information System (DIS) with data dating back to the mid 1990s. A total of 116 unique navigation channel maintenance dredging jobs were identified as seen in Figure 2.

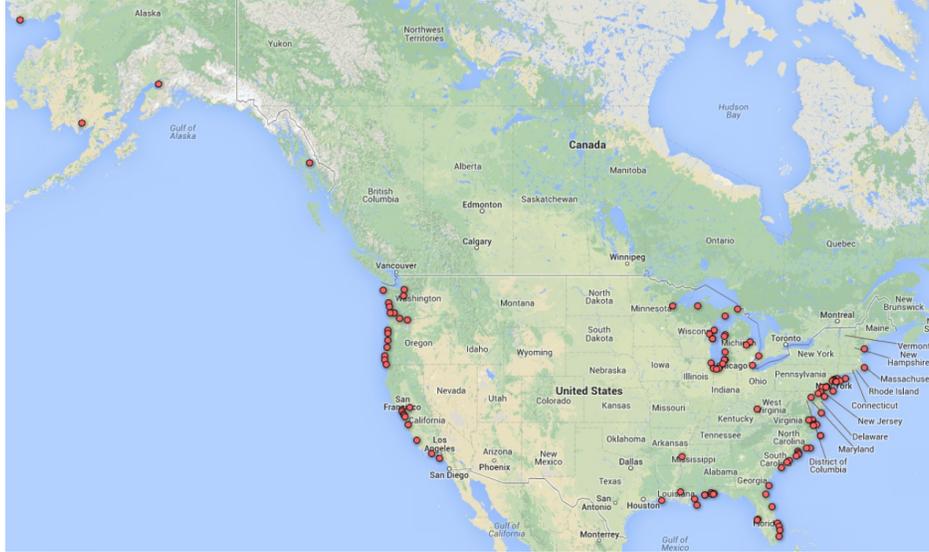


Figure 2: Graphical Depiction of 116 Dredge Jobs Locations

The dredging jobs volumes and costs are shown in Table A1. These values were calculated by averaging over the range of years for which DIS data was available for each job [9]. An average of 416,427 cubic yards, with a standard deviation of 702,096 cubic yards, was dredged across 116 jobs. The largest dredging job considered averaged 5.4 million cubic yards and the smallest job considered in the set had an average of 4,376 cubic yards dredged each year. From a dredging cost perspective, the most expensive job in the pool considered was \$14,477,345, while the minimum cost was \$46,440. The average expenditure per job was \$1,922,517, with a standard deviation of \$2,444,404. All type of costs associated with dredging jobs from start to finish including the mobilization/demobilization, fuel, labor, maintenance, and etc. are considered in calculation of each job cost.

The DIS historical data was also used by Nachtmann et al. [9] to gather information on performance data for the individual Corps-owned dredge vessels, as well as the dredging companies performing contract work for the USACE. Numerous dredging jobs conducted by 30 different companies, over more than a decade, were considered in order to obtain representative daily production rates. As emphasized in [9], the statistical average of dredge vessel production rates was derived over specific years (see Table A2) and therefore should not be interpreted as baseline for any individual dredging vessel in the Corps or industry fleet.

For the 116 jobs considered, a total of 130 unique restricted periods were identified and used within the (CP-DS) optimization model. The number of unique restricted periods exceeds the number of dredging jobs because in some instances. As explained in [9], these RPs were identified using the USACE Threatened, Endangered, and Sensitive Species Pro-

tection and Management System. The average length of all RPs considered was 143.6 days, with a standard deviation of 71.2 days. Table 2 summarizes the types of restricted periods considered by the (CP-DS) model.

Table 2: Summary of Restricted Periods (RPs) (duration: days)

<b>RP Type</b>	<b>Total Duration</b>	<b>Avg. Duration</b>	<b>No. of Jobs with RP</b>
Fish	12,541	187	67
Marine Turtles	5,773	222	26
Birds	3,221	179	18
Marine Mammals	3,006	137	22
Crustaceans	1,496	150	10
Marine Mussels	832	104	8
<b>TOTAL:</b>	<b>26,869</b>	<b>178</b>	<b>151</b>

The distance between jobs was used to calculate travel time of dredge vessels. A from-to distance matrix was constructed by using a GIS layer that computed travel distance on the waterways between all prospective job locations. The (CP-DS) model assumed an average travel rate of 50 miles per day for dredge vessels moving between jobs.

## 3.2 Extension and Improvement

In the following sections, the expansion and improvements to the CP-DS model are discussed individually. In Section 4.1, a comprehensive model which incorporates all the modifications is presented. All computational experiments are conducted on real test instance with 116 dredging jobs, 30 available dredge vessels and 138 restricted periods over one year time horizon. The CP-DS model will be considered as our *base model*. The base model will be compared with modified models in the following sections and shown using a gray highlighted color in the tables. The dredge vessel scheduling problem formulations in Section 2.2 are modeled in IBM ILOG CPLEX Optimization Studio 12.3 [4], which uses IBM ILOG CPLEX 12.3 to solve MIP and IBM ILOG CP Optimizer 12.3 to solve CP models. All test problems are run on a Core(TM) i7 CPU @ 2.93 GHz, 8 GB RAM computer.

### 3.2.1 Allowing Partial Dredging During Restricted Periods

The Corps describes environmental windows as “temporal constraints placed upon the conduct of dredging or dredged material disposal operations in order to protect biological resources or their habitats from potentially detrimental effects” [1]. The scheduling of environmental work windows is intended to minimize environmental impacts by limiting dredging

activities to time periods when biological resources are not present or are least sensitive to disturbance. Surveys conducted by the Corps indicate that approximately 80% of all O&M dredging projects are subject to some form of environmental work window constraint, with wide variations across districts. The Atlantic and Pacific Coast districts report the highest percent of projects with restrictions (up to 100%) and the districts in the Gulf of Mexico and Mississippi Valley regions report the lowest percentage (less than 20%) [1]. Our data analysis on the restricted periods summarized in Table 2 supports the results of this survey and shows 6 dredging jobs cannot be conducted in any time of year because of the restricted periods associated with these jobs. For this reason, it is important to be able to consider so-called partial dredging. That is, only dredging a certain % of capacity during a restricted period.

Restricted period relaxations can only be implemented in localized areas after extensive research has been conducted to pinpoint species migratory patterns and sensitivities to dredging activities. In this research we used partial dredging during restricted periods as the relaxation and studied the impact of relaxing each window individually, as well as collectively. Partial dredging during a restricted period means a dredge vessel can work during the restricted periods, but with a slower operation rate. Having encountered a group of aquatic animals like turtles, fish and other wildlife during the restricted periods, dredging vessels might have to stop dredging to prevent causing harm.

In order to modify the model to allow partial dredging during the restricted periods, the intensity function of the interval variable  $z_j$  in Section 2.2 is changed and set in the range of (0%, 100%) exclusive. The work restriction during the RPs mostly depends on the type of living animals in the area. These values can be obtained by using the historical dredging data and averaging over the range of years for which the partial dredging data was available for each job.

As mentioned in Section 2.2, an interval variable represents an interval of time during which something happens. An important feature of interval variables is the fact that they can be optional, which means they can be *present* in the solution or *absent*. An interval is characterized by a possible *start* and *end* value and a *size*. The *length* of an interval variable is defined as its end value minus its start value [3], which is equal to the size of the interval variable if the intensity function associated with that variable is 100% between its start and end time. The size of an interval variable can be interpreted as the work requirements of the variable. For example, suppose an employee works for 5 days a week full time and does not work on weekends. As shown in Figure 3 (a) the intensity function of worker 1 is 100% for the first 5 days of the week and 0% for the weekends. The length of his work ( $6 - 1 = 5$ ) is equal to the size of his work which is 5 man-days in one week time horizon. On the other

hand, worker 2 (Figure 3 (b)) works full time in first four days of a week and then take one day off and works half time in the weekends. His work length is 7 ( $8 - 1 = 7$ ) days, but he delivers 5 man-day work size, which is the same as worker 1. Likewise, the intensity function of each dredging job can be manipulated in order to allow for the appropriate amount of partial dredging during the restricted periods.

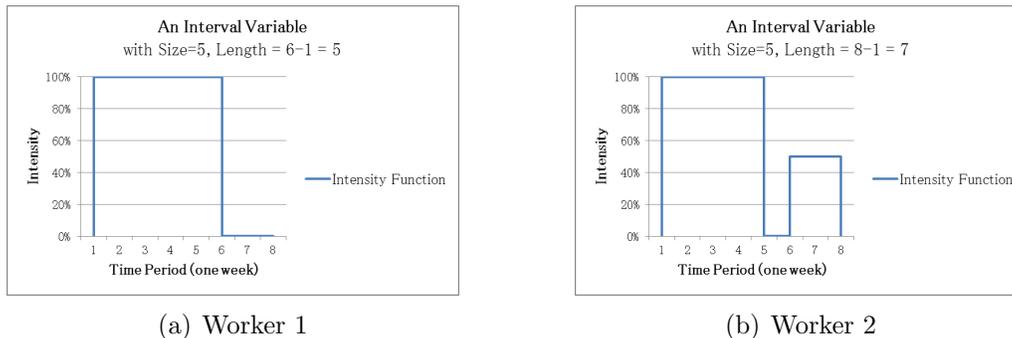


Figure 3: Two Interval Variables with the Same Sizes and Different Lengths

The intensity function of each dredging job can be set to an amount less than 100% during the restricted period due to the type of wildlife living in or migrating to the location of the job in the restricted periods. This amount can vary by each job-vessel combination according to a different system of monitoring, stop time and setup time of each vessel. In order to study the impact of each individual restricted period on the solutions, the intensity function of the interval variable job associated with the restricted period is set to 50% functional. This means we can dredge the job at 50% operation rate instead of 100% during the normal working days. The result of this relaxation is shown in Table 3. By allowing dredging in restricted periods, we should be able to improve the quality of our solution and increase the total cubic yards of dredging with the same available resources during and budget restriction. However, this relaxation expands the solution space of the model and consequently might have negative impact on the results, especially when we relax the restricted periods associated with the jobs that we already dredged in our base model. In Table 3, the percentage change from relaxing an individual restricted period versus the full restriction implemented in the base model is provided. Notice, in some cases this additional flexibility improved the objective. In other cases, the flexibility resulted in a search space so large that an inferior solution is obtained in the amount of time allotted. As mentioned, the best solution of the base model with no relaxation is shown in highlighted gray in the table.

Table 3: Individual Impacts of Restricted Periods Relaxation to 50% Dredging (solution time: sec; dredge, travel, idle time: days)

<b>Relaxed RP</b>	<b>Obj. Function</b>	<b>Sol. Time</b>	<b>Dredge Time</b>	<b>Travel Time</b>	<b>Idle Time</b>	<b>Improv.</b>
5	30,632,870	609	3,217	2,257	934	-0.4%
23	30,632,870	609	3,717	2,213	765	-0.4%
50	30,632,870	609	4,728	1,859	790	-0.4%
32	30,745,673	608	3,807	2,217	767	-0.1%
71	30,760,420	608	3,988	3,210	982	0.0%
⋮	⋮	⋮	⋮	⋮	⋮	⋮
-	30,764,006	610	4,759	2,301	571	0.0%
⋮	⋮	⋮	⋮	⋮	⋮	⋮
36	31,325,902	607	5,031	1,738	1,258	1.8%
106	31,402,795	608	4,857	2,270	610	2.1%
58	31,662,392	608	4,977	2,234	961	2.9%
6	32,570,619	608	3,332	2,021	548	5.9%
17	33,433,044	607	5,162	2,464	987	8.7%

In Table 3, dredge time is the total time that dredge vessels spend on operating the jobs. Travel time is the sum of the time that each dredge vessel spend traveling between two consecutive jobs for all dredge vessels. The travel time between the origin location and each job's location is not considered in total travel time. Idle time is the total time of all dredge vessels being idle. A dredge vessel becomes idle when it finishes a job and travels to another job but cannot start dredging the job because the restricted period(s) of that job will not allow. Therefore, it will stay idle until the restricted period(s) have passed. As shown in Table 3, the largest improvement in objective function (8.7%) is obtained by allowing 50% of dredging during the restricted period number 17 with the range [91, 334] (in time horizon of a year [1, 365]), which is associated with the marine turtle restriction on the dredging job in the LA Calcasieu River Bar Channel.

We know that the solution of the model with no dredging allowed during the restricted periods (the base model solution) is feasible to the relaxed models with restricted periods that can be dredged. In order to overcome the negative impact of expanding our solution space on the objective functions of our relaxed model, we set the base solution as the starting point to the relaxed models. The results of feeding the base model solution to the relaxed models as the starting point to search in CP is shown in Table 4. As we see, there is no negative improvement in any of the solutions and all the solutions are at least as good as the base solution.

Table 4: Individual Impacts of Restricted Periods Relaxation to 50% Dredging after Feeding Initial Solution to the Problems (solution time: sec; dredge, travel, idle time: days)

<b>Relaxed RP</b>	<b>Obj. Function</b>	<b>Sol. Time</b>	<b>Dredge Time</b>	<b>Travel Time</b>	<b>Idle Time</b>	<b>Improv.</b>
none	30,764,006	610	4,759	2,301	571	0.0%
1	30,764,006	608	4,720	2,430	650	0.0%
2	30,764,006	608	4,759	2,234	706	0.0%
3	30,764,006	608	4,758	2,349	897	0.0%
4	30,764,006	608	4,758	2,212	670	0.0%
5	30,764,006	606	4,759	2,118	948	0.0%
7	30,764,006	606	4,761	2,528	570	0.0%
8	30,764,006	607	4,759	2,109	903	0.0%
9	30,764,006	607	4,757	2,292	701	0.0%
10	30,764,006	606	4,757	2,292	701	0.0%
11	30,764,006	607	4,757	2,292	701	0.0%
12	30,764,006	606	4,757	2,292	701	0.0%
13	30,764,006	607	4,757	2,312	750	0.0%
14	30,764,006	607	4,722	2,315	922	0.0%
15	30,764,006	608	4,722	2,315	922	0.0%
16	30,764,006	608	4,720	2,430	650	0.0%
19	30,764,006	608	4,757	2,292	701	0.0%
20	30,764,006	608	4,716	2,227	854	0.0%
21	30,764,006	608	4,757	2,292	701	0.0%
22	30,764,006	609	4,758	2,159	714	0.0%
23	30,764,006	608	4,759	2,234	706	0.0%
24	30,764,006	608	4,757	2,292	701	0.0%
25	30,764,006	609	4,718	2,287	838	0.0%
26	30,764,006	608	4,759	2,412	583	0.0%
27	30,764,006	608	4,735	2,344	810	0.0%
28	30,764,006	609	4,758	2,568	706	0.0%
29	30,764,006	609	4,759	2,234	706	0.0%
30	30,764,006	608	4,757	2,377	767	0.0%
31	30,764,006	608	4,717	1,933	933	0.0%
32	30,764,006	609	4,757	2,292	701	0.0%

Continued on the next page

Table 4 – continued from previous page

<b>Relaxed RP</b>	<b>Obj. Function</b>	<b>Sol. Time</b>	<b>Dredge Time</b>	<b>Travel Time</b>	<b>Idle Time</b>	<b>Improv.</b>
34	30,764,006	608	4,757	2,292	701	0.0%
38	30,764,006	608	4,758	2,421	645	0.0%
39	30,764,006	608	4,759	2,383	661	0.0%
41	30,764,006	608	4,736	2,295	917	0.0%
42	30,764,006	609	4,758	2,421	645	0.0%
43	30,764,006	609	4,757	2,292	701	0.0%
44	30,764,006	610	4,757	2,292	701	0.0%
45	30,764,006	610	4,757	2,292	701	0.0%
46	30,764,006	609	4,757	2,292	701	0.0%
47	30,764,006	609	4,759	2,167	1,048	0.0%
48	30,764,006	610	4,722	2,315	922	0.0%
49	30,764,006	609	4,757	2,292	701	0.0%
50	30,764,006	609	4,757	2,292	701	0.0%
51	30,764,006	609	4,742	2,251	738	0.0%
52	30,764,006	609	4,757	2,292	701	0.0%
53	30,764,006	609	4,758	2,349	897	0.0%
54	30,764,006	609	4,776	2,556	699	0.0%
55	30,764,006	609	4,757	2,292	701	0.0%
56	30,764,006	608	4,740	2,166	1,080	0.0%
57	30,764,006	609	4,757	2,292	701	0.0%
59	30,764,006	608	4,759	2,378	659	0.0%
60	30,764,006	609	4,736	2,276	982	0.0%
61	30,764,006	608	4,733	2,150	965	0.0%
62	30,764,006	610	4,757	2,292	701	0.0%
63	30,764,006	609	4,757	2,292	701	0.0%
64	30,764,006	609	4,722	2,315	922	0.0%
65	30,764,006	607	4,757	2,449	721	0.0%
66	30,764,006	609	4,757	2,292	701	0.0%
67	30,764,006	608	4,757	2,292	701	0.0%
68	30,764,006	611	4,722	2,315	922	0.0%
69	30,764,006	610	4,722	2,315	922	0.0%
70	30,764,006	610	4,757	2,292	701	0.0%
72	30,764,006	610	4,760	2,271	858	0.0%
73	30,764,006	611	4,757	2,292	701	0.0%
74	30,764,006	610	4,757	2,292	701	0.0%

Continued on the next page

Table 4 – continued from previous page

<b>Relaxed RP</b>	<b>Obj. Function</b>	<b>Sol. Time</b>	<b>Dredge Time</b>	<b>Travel Time</b>	<b>Idle Time</b>	<b>Improv.</b>
75	30,764,006	611	4,758	2,421	645	0.0%
76	30,764,006	610	4,736	2,276	982	0.0%
77	30,764,006	610	4,757	2,459	922	0.0%
78	30,764,006	609	4,757	2,292	701	0.0%
79	30,764,006	609	4,759	2,055	855	0.0%
80	30,764,006	611	4,757	2,292	701	0.0%
81	30,764,006	609	4,757	2,292	701	0.0%
82	30,764,006	610	4,758	2,255	790	0.0%
83	30,764,006	610	4,758	2,349	897	0.0%
84	30,764,006	609	4,757	2,292	701	0.0%
85	30,764,006	609	4,757	2,449	721	0.0%
86	30,764,006	610	4,757	2,476	845	0.0%
87	30,764,006	610	4,757	2,292	701	0.0%
88	30,764,006	609	4,716	2,269	853	0.0%
89	30,764,006	609	4,759	2,234	706	0.0%
90	30,764,006	609	4,757	2,292	701	0.0%
91	30,764,006	611	4,757	2,292	701	0.0%
92	30,764,006	610	4,757	2,292	701	0.0%
93	30,764,006	610	4,757	2,292	701	0.0%
94	30,764,006	611	4,757	2,292	701	0.0%
95	30,764,006	609	4,757	2,292	701	0.0%
96	30,764,006	611	4,757	2,292	701	0.0%
97	30,764,006	613	4,757	2,292	701	0.0%
98	30,764,006	609	4,757	2,292	701	0.0%
99	30,764,006	612	4,757	2,292	701	0.0%
100	30,764,006	611	4,757	2,292	701	0.0%
101	30,764,006	611	4,757	2,292	701	0.0%
102	30,764,006	611	4,757	2,292	701	0.0%
103	30,764,006	609	4,757	2,292	701	0.0%
104	30,764,006	611	4,757	2,292	701	0.0%
105	30,764,006	610	4,757	2,292	701	0.0%
107	30,764,006	611	4,716	2,389	837	0.0%
108	30,764,006	611	4,758	2,292	650	0.0%
109	30,764,006	610	4,758	2,510	562	0.0%
111	30,764,006	611	4,759	2,087	833	0.0%

Continued on the next page

Table 4 – continued from previous page

<b>Relaxed RP</b>	<b>Obj. Function</b>	<b>Sol. Time</b>	<b>Dredge Time</b>	<b>Travel Time</b>	<b>Idle Time</b>	<b>Improv.</b>
112	30,764,006	611	4,757	2,049	864	0.0%
113	30,764,006	611	4,758	2,349	897	0.0%
114	30,764,006	610	4,758	2,324	531	0.0%
115	30,764,006	611	4,757	2,292	701	0.0%
116	30,764,006	612	4,758	2,421	645	0.0%
117	30,764,006	610	4,757	2,292	701	0.0%
118	30,764,006	610	4,716	2,408	714	0.0%
119	30,764,006	610	4,777	2,488	942	0.0%
120	30,764,006	611	4,757	2,292	701	0.0%
121	30,764,006	611	4,757	2,292	701	0.0%
122	30,764,006	610	4,757	2,292	701	0.0%
123	30,764,006	611	4,757	2,292	701	0.0%
124	30,764,006	611	4,759	2,118	948	0.0%
125	30,764,006	612	4,757	2,292	701	0.0%
126	30,764,006	612	4,758	2,192	928	0.0%
127	30,764,006	611	4,758	2,175	586	0.0%
128	30,764,006	611	4,757	2,292	701	0.0%
129	30,764,006	611	4,757	2,292	701	0.0%
130	30,764,006	612	4,757	2,292	701	0.0%
131	30,764,006	611	4,757	2,292	701	0.0%
132	30,764,006	611	4,757	2,292	701	0.0%
133	30,764,006	614	4,733	2,478	613	0.0%
134	30,764,006	612	4,757	2,292	701	0.0%
135	30,764,006	612	4,733	2,478	613	0.0%
136	30,764,006	613	4,753	2,355	617	0.0%
137	30,764,006	613	4,757	2,292	701	0.0%
138	30,764,006	611	4,718	2,287	838	0.0%
18	30,779,710	609	4,836	2,526	980	0.1%
71	30,779,710	610	5,184	2,256	685	0.1%
110	30,779,710	611	4,163	2,895	1,046	0.1%
35	30,883,336	608	4,256	2,691	1,089	0.4%
33	30,911,249	608	4,926	2,485	849	0.5%
36	31,255,264	610	4,445	2,462	978	1.6%
40	31,269,833	608	3,755	2,514	1,029	1.6%
37	31,346,116	609	4,028	2,944	765	1.9%

Continued on the next page

Table 4 – continued from previous page

<b>Relaxed RP</b>	<b>Obj. Function</b>	<b>Sol. Time</b>	<b>Dredge Time</b>	<b>Travel Time</b>	<b>Idle Time</b>	<b>Improv.</b>
106	31,490,782	611	4,928	2,606	896	2.4%
58	31,550,350	609	4,191	2,606	835	2.6%
6	32,658,606	607	4,318	2,507	1,727	6.2%
17	33,433,044	609	3,935	2,536	690	8.7%

Similar to Table 3, the largest improvement in objective function in Table 4 is obtained by allowing 50% of dredging during restricted period number 17 by the same amount of 8.7%.

Another interesting question in studying the impact of restricted periods on the solutions is what happens when we allow partial dredging during all restricted periods. The impacts of all restricted periods of dredging jobs by allowing to dredge at different percent of relaxation from 0% to 50% with 5% increment is shown in Table 5. In implementing the problem instances, the budget limitation has been removed from the model to focus on the impacts of restricted periods. The total cubic yards of dredging jobs is 48,305,584 cubic yards, which can be obtained by allowing 51% of dredging in all RPs as shown in the last row of Table 5 (highlighted green).

Table 5: All Restricted Periods Relaxation Impacts on the Objective Function without Overall Budget Limits (objective function: cubic yards dredging, duration: days)

<b>Relax.</b>	<b>Obj. Function</b>	<b>Sol. Time</b>	<b>Dredge Time</b>	<b>Travel Time</b>	<b>Idle Time</b>	<b>Improv.</b>
0%	31,145,977	606.5	4,143	1,319	1,233	0.0%
5%	34,504,954	607.2	7,605	2,104	148	10.8%
10%	35,173,379	608.8	8,137	1,848	6	12.9%
15%	39,008,287	608.8	8,195	2,094	0	25.2%
20%	39,585,711	608.0	8,976	1,442	0	27.1%
25%	40,419,016	608.2	9,534	886	11	29.8%
30%	40,419,016	608.4	8,440	962	0	29.8%
35%	40,419,016	610.2	9,170	839	0	29.8%
40%	42,891,619	609.2	9,290	676	161	37.7%
45%	42,891,619	609.3	9,649	642	0	37.7%
50%	42,891,619	608.9	7,748	722	0	37.7%
51%	48,305,584	610.2	9,276	1,075	25	55.1%

Similar to the results in Table 4, we set the base model solution as the starting point for the solutions obtained in Table 5. However, the results are almost the same as the problem

without starting solutions, as shown in Table 6.

Table 6: All Restricted Periods Relaxation Impacts on the Objective Function without Overall Budget Limits after Feeding Initial Solution to the Problems (objective function: cubic yards dredging, duration: days)

Relax.	Obj. Function	Sol. Time	Dredge Time	Travel Time	Idle Time	Improv.
0%	31,145,977	606.0	4,143	1,319	1,233	0.0%
5%	34,504,954	609.6	7,766	1,794	201	10.8%
10%	35,750,803	609.4	8,539	1,896	14	14.8%
15%	38,641,294	609.5	7,733	1,698	116	24.1%
20%	40,419,016	608.9	8,141	1,954	99	29.8%
25%	40,419,016	607.3	8,085	1,851	222	29.8%
30%	40,419,016	608.4	8,158	1,833	305	29.8%
35%	40,419,016	608.6	7,760	2,171	151	29.8%
40%	42,891,619	608.2	8,741	919	227	37.7%
45%	42,891,619	609.0	9,637	593	0	37.7%
50%	42,891,619	609.3	6,373	1,932	612	37.7%
51%	48,305,584	608.0	9,278	1,319	42	55.1%

Figure 4 demonstrates the increment in total cubic yards of dredging from the base model with no dredging to partial dredging of restricted periods at 51%. The improvement in objective function of total cubic yards of dredging in comparison with the base model (first row of the table) is shown in the last column of the previous table.

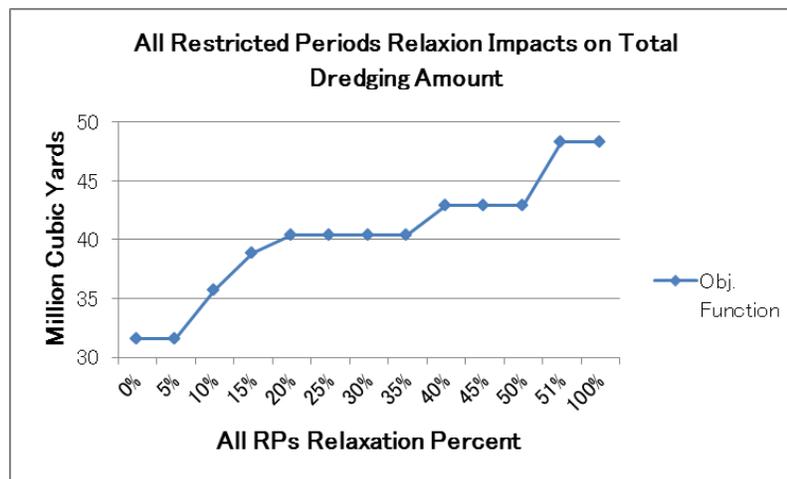


Figure 4: All Restricted Periods Relaxation Impacts on Total Cubic Yards of Dredging

Figure 5 shows the increment in total dredging time and decrement in total travel time and idle time by increasing the relaxation of dredging in restricted periods from 0% (base

model) to 51%.

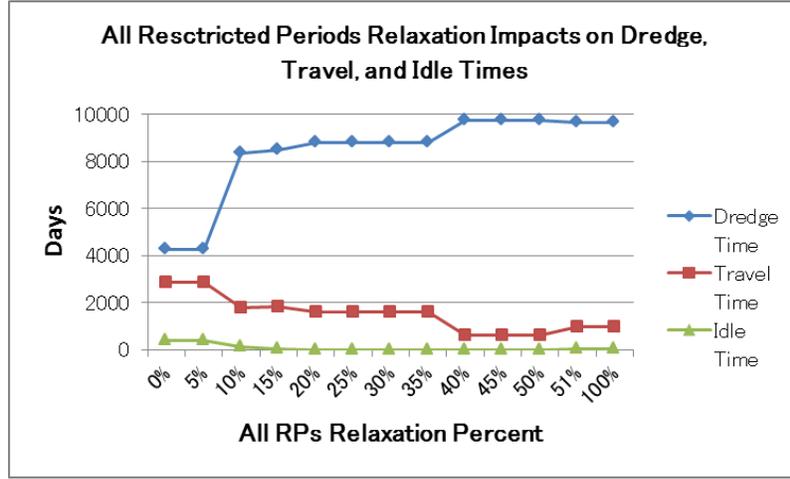


Figure 5: All Restricted Periods Relaxation Impacts on Total Dredge, Travel, and Idle Time

As shown in Figure 5, by allowing more dredging in the restricted periods the total amount of travel and idle time will decrease and the vessels spends more time on dredging their assigned jobs.

### 3.2.2 Variable Job Sizes

The CP-DS model algorithm attempts to complete a job before moving on. However, in reality, there is often a “minimum requirement” that must be met as well as a “target requirement” that would be ideal to achieve if time/money allows. In this context, the size of the jobs are now variable and may be chosen from the range [minimum requirement, target requirement] inclusive. In order to modify the CP-DS model to take into account the variable job sizes we add these two parameters and constraints to the model in Section 2.2:

#### Additional Parameters:

- $h_{jd}$ , the target requirement of dredging job  $j \in J$  using vessel  $d \in D$ ,
- $m_{jd}$ , the minimum requirement of dredging job  $j \in J$  using vessel  $d \in D$ .

#### Additional Constraints:

$$\text{SizeOf}(y_{jd}) \leq h_{jd} \quad j \in J, d \in D \quad (16)$$

$$\text{SizeOf}(y_{jd}) \geq m_{jd} \times \text{PresenceOf}(y_{jd}) \quad j \in J, d \in D \quad (17)$$

Constraints (16) ensure that the size of all dredging job  $j \in J$  conducted by dredge vessel  $d \in D$  remains less than or equal to the target size of the jobs. Similarly, Constraints 17 make sure that the size of all dredging jobs  $j \in J$  are greater than or equal to the minimum job size for all available dredge vessels  $d \in D$  if the variable  $y_{jd}$  is present in our solution. As we mentioned in Section 2.2 and discussed in Section 3.2.1, an interval variable can be optional. This means these variables can be *present* in the solution or *absent*. If we do not include the `PresenceOf( $y_{jd}$ )` term in Constraints 17, the optional interval variables  $y_{jd}$  for all  $j \in J, d \in D$  will receive a positive size and accordingly will be presented in the solution. This means each job  $j \in J$  will be conducted by all available dredge vessel  $d \in D$ .

Table 7 shows the total amount of dredging and total dredge, travel, and idle time for our base model (gray highlighted row) with the original size of jobs equal to the target requirements and compares it with four other test instances with different ranges of [minimum requirement, target requirement] for all jobs. In the first column of the table,  $h$  is the original size of the jobs in our base model with constant job sizes. In all four test instances, we set the target size of jobs equal to their original size and a fraction of the target size as the minimum requirements of the jobs. For example, the job size range  $[0, h]$  means the minimum requirements of all jobs are equal to 0 and their target requirements are equal to the original size of the jobs. The range  $[0.25h, h]$  means the minimum requirements of all jobs and their target requirements are equal to 25% of the original sizes ( $h$ ) and the original size of the jobs, respectively.

Table 7: Impact of Having Variable Jobs Size with Different Range [min. req., target req.] (objective function: cubic yards dredging, duration: days)

<b>Job Size Range</b>	<b>Obj. Function</b>	<b>Sol. Time</b>	<b>Dredge Time</b>	<b>Travel Time</b>	<b>Idle Time</b>	<b>Gap%</b>
$h$	30,764,006	609.5	4,759	2,301	571	0%
$[0, h]$	12,628,669	611.5	1,376	2,954	2,364	-59%
$[0.25h, h]$	19,188,239	612.4	1,768	1,994	910	-38%
$[0.50h, h]$	22,562,323	610.6	2,734	3,209	1,130	-27%
$[0.75h, h]$	28,508,171	611.6	2,934	2,846	1,096	-7%

As we can see in the Table 7 allowing variable job sizes have a negative impacts on the objective function of total cubic yards of dredging. This is because the solution space grows significantly. According to the gap column in the table, the tighter the range of job requirement, the better the solution that can be found by CP Optimizer. As shown in Figure 6, the gap between the objective function of our base model and the variable job size model

decreases from  $-59\%$  to  $-7\%$  if we can assess the dredging jobs requirement more precisely. This result suggests that (i) decision-makers should be precise in determining the range of job production and (ii) the optimizer should establish a baseline solution (using base model) from which CP can begin its search with the constraints added in this section.

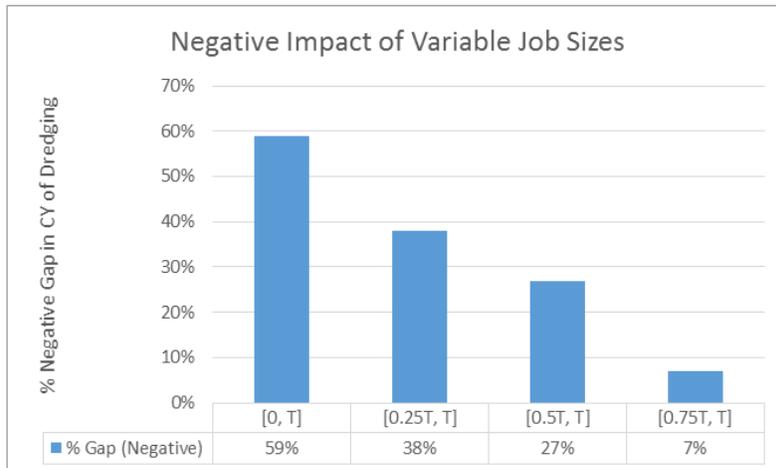


Figure 6: Impact of Allowing Variable Job Sizes

### 3.2.3 Multiple Trips to the Same Job

Some jobs are may not be done within one environmental work window. For example, a government dredge can operate up to 15 days in May, but then not dredge again until June 15 because of the restricted period between May and June 15. In order to modify the model to allow multiple trips to the same jobs (even with different dredge vessels), each job is split to some subtask according to the restricted periods of the job. For example, if job 1 has the restricted period of  $I(1) = [152, 274]$ , it will be split to 2 subtasks in such a way that subtask 1 can only take place in the range  $[1, 151]$  and subtask 2 in the range  $[275, 365]$ , which are the allowed ranges of dredging for job 1. In order to modify the CP-DS model in Section 2.2, additional sets, variables and constraints are used. Note that similar to the Section 3.2.2, the job sizes are also variable in the modified model and the CP optimization tool will decide the amount of dredging for each job to maximize the total cubic yards of dredging.

#### Additional Sets:

- $i \in I_j^c$ , set of possible subtasks of each dredge job  $j \in J$ . These subtasks can start and finish outside of job  $j \in J$  restricted periods,  $W_j$ , wherever  $I(j) \neq 0$ .

**Modified and Additional Variables:**

- $y_{jid}$ , optional interval variable when subtask  $i$  of job  $j \in J$  is assigned to dredge vessel  $d \in D$ .
- $Y_{ji} = \{y_{ji1}, y_{ji2}, \dots, y_{jiD}\}$ , set (array) of interval variables representing possible dredge vessel  $d$  that can be assigned to subtask  $i$  of job  $j \in J$ .
- $Y_d = \{y_{11d}, y_{12d}, \dots, y_{21d}, y_{22d}, \dots, y_{J1d}, y_{J2d}, \dots\}$ , set (array) of interval variables representing possible subtask  $i$  of job  $j \in J$  that can be assigned to dredge vessel  $d \in D$ .
- $x_{ji}$ , optional interval variable associated with subtask  $i$  of job  $j \in J$ .

After we split each job to some subtask in the allowed range of dredging for the job, we do not need the  $\text{ForbidExtend}(z_j, I(j)), \forall j \in J$  constraints because we already set the start and end time of each subtask to a time period in which dredging is allowed.

**Removed Constraints:**

$$\text{ForbidExtend}(z_j, I(j)) \quad j \in J \quad (18)$$

In addition to the variable job size constraints discussed in Section 3.2.2, the following constraints are added to the CP-DS model.

**Modified and Additional Constraints:**

$$\text{Span}(z_j, x_{ji}) \quad j \in J \quad (19)$$

$$\text{Alternative}(x_{ji}, Y_{ji}) \quad j \in J, i \in I_j^c \quad (20)$$

$$\text{NoOverlap}(Y_d, TD[\text{Type}(j), \text{Type}(j')]) \quad d \in D \quad (21)$$

Constraints (19) state that each interval variable  $z_j, j \in J$  spans over all present interval variables from the set  $\{x_{j1}, x_{j2}, \dots\}$ . The interval variable  $z_j$  starts with the first present interval from  $\{x_{j1}, x_{j2}, \dots\}$  and ends with the last job in the set that ends in the time horizon. Note that, similar to Section 3.2.2, we still have the constraints for target requirement and minimum requirement of all jobs. The sum of all subtasks of job  $j \in J$  is still between the range of  $[m_j, h_j]$ . Similar to Constraints (9) and (15) of CP-DS model in Section 2.2, Constraints (20) and (21) are for the assignment of available dredge vessels and the travel

time between jobs, respectively. Constraints (20) make the CP choose exactly one possible assignment for  $x_{ji}$  from all possible assignments. In scheduling of the dredge fleet we need to consider the time to travel between two consecutive jobs locations operated by the same dredge which is handled by Constraints (21).

Table 8 shows the computational results from allowing multiple trips to the same job in comparison with our base model with 116 jobs, 30 dredge vessels, and 130 restricted periods.

Table 8: Impact of Allowing Multiple Trips to the Same Jobs (objective function: cubic yards dredging, duration: days)

Model	Job Size Range	Obj. Function	Sol. Time	Dredge Time	Travel Time	Idle Time	Improv.
Base Model	$h$	30,764,006	609.5	4,759	2,301	571	-
Var. Job Sizes	$[0.25h, h]$	19,188,239	612.4	1,768	1,994	910	0%
Multiple Trips	$[0.25h, h]$	29,915,071	622.9	4,124	3,414	2,378	56%

As we can see in the Table 8, the total cubic yards of dredging by allowing multiple trips to the same jobs is about 3% less than our base model in which a dredge vessel could not start a job without finishing it before its restricted time period(s). The reason of this increment in total cubic yards of dredging is that in the multiple trips model the size of each job is variable. As discussed in Section 3.2.2, having variable job sizes causes expansion in the solution space which has a negative impact on the objective function of our model. However, we could improve the objective function by 56% comparing with the variable job sizes model in Section 3.2.2 with the data set utilizing the range of job sizes of  $[0.25h, h]$  in Table 7. Put simply, the ability to visit sites multiple times yields extensive benefit to the corps (especially when variable job size is not included).

### 3.2.4 Multiple Dredges on the Same Job

The ability to dredge a job with multiple dredge vessels in the model can speed up the dredging process of jobs and maximize the utilization of available dredge vessels. Moreover, in practice, some jobs need to be done by multiple dredge vessels depending on geographic characteristics of the job locations and depth and wideness of the waterways. For example, Clearwater Harbor, Florida needs Yaquina, Essayons, and contract dredges typically. If these jobs can work interdependently in separate areas, the problem would be easy to handle. Such dredging jobs can be split into additional jobs with associated location, size, and compatible dredge vessel type. On the other hand, if all different types of dredge vessels should work together simultaneously and start and end the dredging job at the same time,

we must modify the Constraints (9) in the CP-DS model (Section 2.2) to make them compatible. The modified constraints are as follows:

**Modified Constraints:**

$$\text{Alternative}(z_j, Y_j, c) \qquad j \in J \qquad (22)$$

Constraints (22) enforce each job  $j \in J$  be assigned to exactly  $c$  dredge vessels from all dredge vessels compatible with the job. The additional parameter  $c$  in *Alternative* global constraints is need to be sure that, if an interval decision variable  $z_j$  is present in the solution, then exactly  $c$  elements of  $Y_j$  must be presented in the solution and all of them start and end together.

The most challenging form of multiple dredges working on a same job is when dredging can happen by any type of dredge vessel with different start and finish times in the time horizon. In order to address this case in our model, similar to Section 3.2.3, each job is divided into a number of subtasks with variable job sizes (as described in Section 3.2.2). In this case, each job is divided to the number of available dredge vessels ( $|D|$ ) in order to maintain the possibility of working all dredge vessels on each job. After solving the problem with the CP optimization tool, the start and end time, size of each subtask and the dredge vessel assigned to each subtask will be determined. Similar to our base model (with one dredge vessel on each job), there is a travel distance between each subtask and it is equal to 0 if two consecutive subtasks are dredged by the same dredge vessel. The total size of each job, which is the summation of all its subtasks, must be less than or equal to the target requirement size of each job. As we divide each job into the number of available dredge vessels subtasks, the following modifications to the sets, variables, and constraints need to be made (similar to Section 3.2.3):

**Additional Set:**

- $k \in D$ , set of dredge subtasks that need to be completed over the planning horizon (each job is divided to  $|D|$  subtasks).

**Modified and Additional Variables:**

- $y_{jkd}$ , optional interval variable when subtask  $k \in D$  of job  $j \in J$  is assigned to dredge vessel  $d \in D$ .
- $Y_{jk} = \{y_{jk1}, y_{jk2}, \dots, y_{jkD}\}$ , set (array) of interval variables representing possible dredge vessel  $d$  that can be assigned to subtask  $k \in D$  of job  $j \in J$ .

- $Y_d = \{y_{11d}, y_{12d}, \dots, y_{1kd}, \dots, y_{1Dd}, y_{21d}, y_{22d}, \dots, y_{2kd}, \dots, y_{2Dd}, \dots, y_{J1d}, y_{J2d}, \dots, y_{Jkd}, \dots, y_{JDd}\}$ , set (array) of interval variables representing possible subtask  $k \in D$  of job  $j \in J$  that can be assigned to dredge vessel  $d \in D$ .
- $x_{jk}$ , optional interval variable associated with subtask  $k \in D$  of job  $j \in J$ .

In addition to variable job size constraints discussed in Section 3.2.2, the following constraints are added to the CP-DS model.

### Modified and Additional Constraints:

$$\text{Span}(z_j, x_{jk}) \quad j \in J, k \in D \quad (23)$$

$$\text{Alternative}(x_{jk}, Y_{jk}) \quad j \in J, k \in D \quad (24)$$

$$\text{NoOverlap}(Y_d, TD[\text{Type}(j), \text{Type}(j')]) \quad d \in D \quad (25)$$

$$\sum_{k \in D} \text{SizeOf}(x_{jk}) \leq h_j \quad j \in J \quad (26)$$

Constraints (23) state that each interval variable  $z_j, j \in J$  spans over all present intervals variables from the set  $\{x_{j1}, x_{j2}, \dots, x_{jD}\}$ . As mentioned in Section 3.2.3, the interval variable  $z_j$  starts with the first present interval from  $\{x_{j1}, x_{j2}, \dots, x_{jD}\}$  and ends with the last one. Similar to Constraints (20) and (21) of the multiple trips model in Section 3.2.3, Constraints (24) and (25) are for the available dredge vessels assignment to the jobs and travel time between jobs enforcement, respectively. Constraints (26) impose a limitation on the total size of each job, which is the summation of the sizes of its subtasks. Note that, similar to Section 3.2.2, the size of each subtask is variable and is less than or equal to the target size (cubic yards of dredging) and greater than or equal to the minimum size of each job.

The ability to multiple dredges work on a same job in our model is the most complex and computationally expensive improvement in our base model. On the other hand, this ability gives us the most possible flexibility to assign jobs to dredge vessels. In this model, the dredge vessels can have multiple trips to the same jobs and dredge any portion of the total size of each job during each visit. By splitting each job into the number of available dredge vessels subtasks, the number of variables and constraints increases enormously and accordingly the solution space will expand dramatically. In order to help the CP optimization tool to find feasible solutions and improve them we removed the budget constraint and also extended the range between the minimum and target requirement of the jobs by setting the minimum requirement to 25% of target requirement size. Without this, CP could not find any feasible solution even for the problem instance with 10 jobs and 5 dredge vessels after

4 hours time limit. An instance with largest number of jobs that could be solved by CP on a Core(TM) i7 CPU @ 2.93 GHz, 8 GB RAM computer was with 57 jobs and 15 dredges. Yet, a problem instance with 57 jobs and 20 dredges ran for 4 hours on the same computer without finding any feasible solution.

Table 9 shows the computational result of allowing multiple dredge vessels working on the same job in comparison with our base model (with at most one dredge assigned to each job) instance tests with 10, 32, and 57 jobs and 5, 10 and 15 dredge vessels. All problem instances ran for four hours or 14,400 seconds.

Table 9: Impact of Allowing Multiple Dredges Working on the Same Jobs with Different Operation Rates and Unit Cost of Dredging (objective function: cubic yards dredging, duration: days, SD: Single Dredge on each Job, MD: Multi Dredges on each Job)

Model	Instance	Obj. Function	Sol. Time	Dredge Time	Travel Time	Idle Time	Improv.
SD	$ J  = 10,  D  = 5$	2,487,676	0.2	347	137	82	0%
MD	$ J  = 10,  D  = 5$	6,743,828	60.5	735	543	313	171%
SD	$ J  = 32,  D  = 10$	8,413,704	3,819.7	864	780	595	0%
MD	$ J  = 32,  D  = 10$	18,922,824	14,409.4	1,873	1,356	398	125%
SD	$ J  = 32,  D  = 15$	8,413,704	3,672.7	909	588	369	0%
MD	$ J  = 32,  D  = 15$	18,150,677	14,451.4	1,667	2,134	1,119	116%
SD	$ J  = 32,  D  = 20$	8,413,704	3,644.8	482	836	336	0%
MD	$ J  = 32,  D  = 20$	10,890,124	14,535.3	1,114	2,832	1,625	29%
SD	$ J  = 32,  D  = 30$	8,413,704	3,620.6	1,101	399	156	0%
MD	$ J  = 32,  D  = 30$	9,469,173	16400.8	1,004	2,467	1,251	13%
SD	$ J  = 57,  D  = 10$	13,044,882	3,647.2	1,176	1,026	351	0%
MD	$ J  = 57,  D  = 10$	21,371,547	14,439.1	2,191	1,086	257	64%
SD	$ J  = 57,  D  = 15$	18,093,069	3,621.0	1,834	717	978	0%
MD	$ J  = 57,  D  = 15$	12,536,703	14,598.6	1,327	2,204	1,499	-31%
SD	$ J  = 57,  D  = 20$	18,093,069	3,611.3	1,932	1,066	989	0%
MD	$ J  = 57,  D  = 20$	no solution	14,400	-	-	-	-%

In this model, by having variable job sizes and multiple dredges working on same jobs there is a trade-off between having more flexibility in assigning dredge vessels to dredging jobs and expanding the solution space enormously. Flexibility gives us more opportunity to dredge more jobs and for each job more cubic yards of dredging and on the other hand solution space expansion makes it hard for CP to find quality solutions.

As we can see in the Table 9, allowing multiple dredges working on same jobs has positive impacts (up to 171%) on the objective function (total cubic yards of dredging) for small instances and negative impacts (-31%) on the big instances (unlike the base model, in the expanded model the instance with 57 jobs and 15 dredges is considered to be a big instance instead of a medium size instance because the number of jobs that need to be scheduled is

$57 \times 15 = 855$  using 15 dredges) because the solution space expands significantly. As will be discussed further in the conclusions of this report, this new modeling paradigm now calls for additional methodological improvements.

### 3.2.5 Different Operation Rates, Unit Cost of Dredging and Budget

The operation of each dredge vessel can vary greatly from one job to another due to wave conditions, weather and sediment types. In order to modify the CP-DS model to work with different operation rates for each vessels in a same job, the parameter  $r_d$  in Section 2.2 is changed to  $r_{jd}, \forall j \in J, d \in D$  and accordingly the parameter  $t_{jd}$ , the time (days) required for dredge vessel  $d$  to complete job  $j$ , will be changed to  $t_{jd} = \lceil q_j / r_{jd} \rceil, \forall j \in J, d \in D$ . The variables and constraints of the CP-DS model in Section 2.2 will remain unchanged.

Separately, the cost of dredging per cubic yard can vary from one dredge vessel to another due to the size of dredge vessels, different types of equipment they use and the crew size of each dredge vessel. Similar to having different operation rates for each job, we can change the parameter  $c_j$ , the cost for completing job  $j$ , in Section 2.2 to  $c_{jd}, \forall j \in J, d \in D$ , the cost for completing job  $j$  by dredge vessel  $d$ .

Finally, in addition to the overall budget constraint on the total cost of all jobs in the base model, we can impose a limitation on the cost of each individual job to not exceed the available budget for each job. The budget constraint for each individual job is as follows:

#### Additional Parameters:

- $b_j$ , the available budget for the job  $j \in J$ .

#### Individual Job Budget Constraints:

$$\text{PresenceOf}(y_{jd}) \times q_j \times c_{ij} \leq b_j \quad j \in J, d \in D \quad (27)$$

Constraints (27) ensure that if job  $j$  is performed by dredge vessel  $d$  ( $\text{PresenceOf}(y_{jd}) = 1$ ), the cost of dredging ( $q_j \times c_{ij}$ ) will not exceed the available budget for job  $j$  ( $b_j$ ). The result of having different operation rates and cost of dredging for each job-dredge combination is shown in Table 10.

Table 10: Impact of Having Different Operation Rates and Cost of Dredging for each Job (objective function: cubic yards dredging, duration: days)

Model	Instance	Obj. Function	Sol. Time	Dredge Time	Travel Time	Idle Time
Base Model	$ J  = 32,  D  = 30$	8,413,704	601.1	1,229	335	93
Different Rates	$ J  = 32,  D  = 30$	8,413,704	601.7	1,101	399	156
Base Model	$ J  = 57,  D  = 30$	17,090,811	602.2	2,565	961	513
Different Rates	$ J  = 57,  D  = 30$	14,907,474	603.0	2,334	810	812
Base Model	$ J  = 116,  D  = 30$	30,764,006	609.5	4,759	2,301	571
Different Rates	$ J  = 116,  D  = 30$	25,141,975	605.8	4,498	1,104	1,520

In Table 10 the result of running the base model and the model with different rate of operation and unit cost of dredging for each job-vessel combination is reported. In this table we are not comparing the base model with the model with different rates because the data structure has been changed and operation rates of each dredge vessel are different from one job to another. Also, we added new budget constraints for each individual job which will have a non-positive impact on the objective function of total cubic yards of dredging.

### 3.2.6 Simulating Downtime for Dredges

In order to simulate the downtime in the model for each dredge on the assigned job(s), as mentioned in Section 3.2.1, the intensity function of the interval variable associated with the specified dredge and all the jobs is changed and set equal to 0%. The interval variables associated with each job-dredge pair exist in the set of variables  $Y_j$  in the CP formulation presented in Section 2.2. By setting the intensity of function of these variable equal to 0%, we tell the CP model that during the downtime period the dredge cannot work at all. We add the following parameters to the model to specify the begin and end of the downtime for each dredge.

#### Additional Parameters:

- $a_d$ , the begin of downtime of dredge  $d$ .
- $b_d$ , the end of downtime of dredge  $d$ .

All that remains is to set the intensity function of the interval variable  $y_{jd}$  (Section 2.2) between the period  $[a_d, b_d]$  equal to 0%,  $\forall j \in J$ .

### 3.2.7 Mob/Demob Cost

Two of the standard ways of calculating the mobilization and demobilization costs is to split the cost among all projects who use the contracted dredge by prorating the costs according to the respective cubic yard of dredging at each job or the amount of travel time/distance for each dredge.

#### Cost based on cubic yards of dredging

In this case we must track the size of all jobs that are performed by a dredge  $d$  during the optimization process of CP. This is done by using  $\text{SizeOf}(y_{jd})$  as discussed in Section 3.2.2. We must add the following constraint to the model to record the total amount of dredging performed by dredge  $d$  on all the jobs.

#### Additional Constraints:

$$\text{Cumulate}_{j \in J}(\text{SizeOf}(y_{jd})) \quad d \in D \quad (28)$$

After calculating the total amount of dredging for each dredge,  $CY_d$ , we can easily specify the proportion of the mob/demob cost for each dredge  $d$ ,  $\text{mob}_d$ , using the following equation:

$$\text{mob}_d = \text{mob}_{\text{total}} \times \frac{CY_d}{CY_{\text{total}}} \quad d \in D \quad (29)$$

#### Cost based on travel time/distance

In the second case, in which we are splitting the mob/demob cost among contract dredges, we need to know how much each dredge travels while dredging different jobs. This is possible by taking advantage of the CP optimizer's ability to formulate a sequence depending setup cost using the  $\text{TypeOfNext}()$  built-in function. We must add the following constraint to get the total travel time/distance of each dredge  $d$ .

#### Additional Constraints:

$$\text{Cumulate}_{j \in J}(TD[\text{Type}(j), \text{TypeOfNext}(Y_d, j)]) \quad d \in D \quad (30)$$

Similar to the first case, after calculating the total travel time/distance for each dredge,  $TR_d$ , we use the following formula to get the proportion of the mob/demob cost for each dredge  $d$ ,  $\text{mob}_d$ .

$$\text{mob}_d = \text{mob}_{\text{total}} \times \frac{TR_d}{TR_{\text{total}}} \quad d \in D \quad (31)$$

### 3.2.8 Dredge Capabilities by Job

In some cases, a particular dredge may not be able to operate on job  $j$ . In order to prevent assigning dredge  $d$  to job  $j$  we can use two different methods. The first method is to set the intensity function associated with interval variable  $y_{jd}$  equal to 0% as we mentioned in Section 3.2.6. In the second method, we set the operation rate  $r_{jd} = 0$  as shown in Section 3.2.5. This means that there is not any increase in objective function, so the CP optimizer will not consider such an assignment.

## 4 Impacts of Implementation

The impact of the implementations in this work can be measured quantitatively, as shown in the remainder of this section. However, of equal importance is the impact of this work on the future of decision analysis within USACE. After initial success with the base model presented at the beginning of this report, maritime professionals were intrigued by the use of operations research to aid in their decision process. However, the potential of the initial tool was met with concern over the fact that many realistic components were not considered. The main impact of this project is that every concern presented by USACE has now been addressed from a modeling perspective. The decision makers now understand that optimization tools can be flexible and extendable and, with the appropriate amount of attention, complex challenges can be modeled. The model that follows in the remainder of this section will be carried into dredge planning meetings on the west and east coast in the next calendar year, ensuring that impacts of our implementation are likely to be shown in increasing numbers in the months to come.

In the remainder of this section we present the comprehensive model containing all the flexibilities and improvements to the base model as described in Sections 3.2.1 through 3.2.8.

### 4.1 The Modified Comprehensive Model

In this section the modified model, MCP-DS, in which all the modifications that we discussed in Sections 3.2.1 through 3.2.4 is presented in a comprehensive formulation. In addition to Section 2.2, the following parameters and variables are used in the MCP-DS formulation.

### Additional Parameters:

- $h_j$ , the target requirement of dredging job  $j \in J$ ,
- $m_j$ , the minimum requirement of dredging job  $j \in J$ ,
- $h_{jd}$ , the target requirement of dredging job  $j$  using vessel  $d$ ,
- $m_{jd}$ , the minimum requirement of dredging job  $j$  using vessel  $d$ .
- $r_{jd}$ , the operation rate (cubic yards/day) of dredge equipment  $d \in D$  conducting job  $j \in J$ ,
- $t_{jd}$ , the time (days) required for dredge vessel  $d$  to complete job  $j$ , and
- $c_{jd}$ , the cost for completing job  $j \in J$  by dredge vessel  $d \in D$ .

### Modified and Additional Decision Variables:

- $y_{jkd}$ , optional interval variable when subtask  $k \in D$  of job  $j \in J$  is assigned to dredge vessel  $d \in D$ .
- $Y_{ji} = \{y_{ji1}, y_{ji2}, \dots, y_{jiD}\}$ , set (array) of interval variables representing possible dredge vessel  $d$  that can be assigned to subtask  $i$  of job  $j \in J$ .
- $Y_d = \{y_{11d}, y_{12d}, \dots, y_{21d}, y_{22d}, \dots, y_{J1d}, y_{J2d}, \dots\}$ , set (array) of interval variables representing possible subtask  $i$  of job  $j \in J$  that can be assigned to dredge vessel  $d \in D$ .
- $x_{jk}$ , optional interval variable associated with subtask  $k \in D$  of job  $j \in J$ .

$$\max \sum_{j \in J} q_j z_j$$

subject to

$$\text{Span}(z_j, x_{jk}) \quad j \in J, k \in D \quad (32)$$

$$\text{Alternative}(x_{jk}, Y_{ji}) \quad j \in J \quad (33)$$

$$\text{NoOverlap}(Y_d, TD[\text{Type}(j), \text{Type}(j')]) \quad d \in D \quad (34)$$

$$\text{Cumulative}(z_j, c_{jd}, B) \quad (35)$$

$$\text{Cumulative}(z_j, 1, |D|) \quad (36)$$

$$z_j.\text{StartMin} = 1 \quad j \in J \quad (37)$$

$$z_j.\text{EndMax} = |T| \quad j \in J \quad (38)$$

$$\sum_{i \in D} \text{SizeOf}(x_{ji}) \leq h_j \quad j \in J \quad (39)$$

$$\text{SizeOf}(y_{jd}) \leq h_{jd} \quad j \in J, d \in D \quad (40)$$

$$\text{SizeOf}(y_{jd}) \geq \text{PresenceOf}(y_{jd}) \times m_{jd} \quad j \in J, d \in D \quad (41)$$

Constraints 32) state that each interval variable  $z_j, j \in J$  spans over all present interval variables from the set  $\{x_{j1}, x_{j2}, \dots, x_{jD}\}$ . As mentioned in Section 3.2.3 the interval variable  $z_j$  starts with the first present interval from  $\{x_{j1}, x_{j2}, \dots, x_{jD}\}$  and ends with the last one. Similar to Constraints (24) of the multiple dredges model in Section 3.2.4, Constraints (33) are included for the assignment of jobs to available dredge vessels. Constraints (34) are in the model for setting the travel time,  $TD[\text{Type}(j), \text{Type}(j')]$ , between two consecutive subtasks  $j$  and  $j'$  that are conducted by the same dredge vessel  $d$ .

Similar to Section 2.2, Constraint (35) imposes that the total cost of dredging job  $j \in J$  by dredge vessel  $d \in D$  with cost of  $c_{jd}$  cannot exceed the total budget  $B$ . Also, Constraint (36) makes sure that the total number of occupied dredge vessels at any time does not exceed the fleet size  $|D|$ . Constraints (37) and (38) are the same as in CP-DS model and set the minimum start time and maximum end time of each job to the first and last day of the planning horizon, respectively. Constraints (39) impose a limitation on the total size of each job, which is the summation of the sizes of its subtasks. Note that similar to Section 3.2.2, the size of each subtask is variable and is less than or equal to the target size (cubic yards of dredging) and greater than or equal to the minimum size of each job. Constraints (40) and (41) ensure that the size of all dredging job  $j \in J$  conducted by dredge vessel  $d \in D$  remain

between the minimum and the target size of the job  $j_m$  if the variable  $y_{jd}$  is present in the solution.

The implementation of our comprehensive model on the relaxation impacts of each individual and entire restricted periods of the MCP-DS model on the different problem instances are shown in Table 11 and 12, respectively.

Table 11: Individual Impacts of Restricted Periods Relaxation to 50% Dredging in MCP-DS model (solution time: sec; dredge, travel, idle time: days)

<b>Relaxed RP</b>	<b>Obj. Function</b>	<b>Sol. Time</b>	<b>Dredge Time</b>	<b>Travel Time</b>	<b>Idle Time</b>	<b>Improv.</b>
121	24,581,333	627	3,922	2,276	199	-14.3%
28	24,880,818	617	3,505	2,773	369	-13.3%
3	25,087,062	613	3,731	2,392	2,089	-12.6%
71	25,236,272	619	4,243	2,417	596	-12.1%
13	25,323,126	614	4,439	1,996	536	-11.8%
48	25,476,675	620	4,227	2,491	762	-11.2%
70	25,509,670	618	4,350	2,390	447	-11.1%
84	25,511,265	624	4,317	2,850	255	-11.1%
67	25,746,209	622	3,828	3,021	754	-10.3%
49	25,818,213	621	4,197	2,491	420	-10.0%
124	25,827,117	619	3,893	2,498	802	-10.0%
136	25,878,293	617	4,286	2,379	1,586	-9.8%
61	25,893,215	620	3,486	2,222	424	-9.8%
116	25,904,376	617	4,119	2,515	584	-9.7%
86	25,910,817	620	4,260	2,557	666	-9.7%
66	25,924,092	616	4,347	2,963	1,335	-9.7%
1	25,969,781	617	4,179	2,972	1,104	-9.5%
56	26,017,505	620	4,657	2,517	618	-9.3%
111	26,058,403	617	3,643	2,438	499	-9.2%
108	26,066,746	614	3,844	2,426	922	-9.2%
109	26,251,266	619	3,819	2,692	569	-8.5%
65	26,305,851	617	4,163	2,312	297	-8.3%
62	26,314,153	622	4,113	1,988	513	-8.3%
37	26,354,590	620	4,538	2,361	648	-8.2%
83	26,364,672	619	4,792	1,154	875	-8.1%
122	26,498,426	622	4,193	2,251	1,790	-7.7%

Continued on the next page

Table 11 – continued from previous page

<b>Relaxed RP</b>	<b>Obj. Function</b>	<b>Sol. Time</b>	<b>Dredge Time</b>	<b>Travel Time</b>	<b>Idle Time</b>	<b>Improv.</b>
125	26,558,505	618	4,096	2,310	546	-7.5%
23	26,576,902	615	4,212	2,616	1,475	-7.4%
68	26,601,860	621	3,596	2,458	565	-7.3%
29	26,642,684	621	4,785	3,141	1,551	-7.2%
72	26,676,879	619	3,708	2,163	1,448	-7.0%
26	26,685,537	615	4,252	2,168	639	-7.0%
8	26,800,327	618	3,803	2,038	2,121	-6.6%
74	26,810,883	614	4,072	2,497	691	-6.6%
51	26,841,316	621	4,097	2,764	1,005	-6.5%
119	26,905,920	616	4,622	1,971	955	-6.2%
15	26,916,603	617	4,305	2,696	1,356	-6.2%
73	26,929,978	616	3,955	1,899	947	-6.2%
77	26,940,160	618	3,945	1,847	2,041	-6.1%
41	27,017,157	618	4,376	2,247	270	-5.9%
134	27,036,656	620	4,132	2,686	1,335	-5.8%
5	27,118,216	624	4,297	2,307	325	-5.5%
128	27,123,344	622	3,924	2,561	806	-5.5%
36	27,235,356	616	4,192	3,017	996	-5.1%
114	27,280,319	619	4,072	2,677	1,157	-4.9%
118	27,304,541	625	4,181	2,869	1,192	-4.9%
117	27,308,351	619	4,113	2,480	1,239	-4.8%
59	27,323,243	618	3,740	2,415	1,446	-4.8%
69	27,434,191	621	4,055	2,204	1,311	-4.4%
60	27,450,821	621	4,019	2,560	1,567	-4.3%
38	27,466,034	612	4,529	2,333	749	-4.3%
39	27,481,385	618	3,664	2,215	1,566	-4.2%
22	27,490,239	619	3,718	2,252	1,696	-4.2%
54	27,524,294	617	4,685	2,234	847	-4.1%
123	27,558,286	624	3,840	2,495	294	-4.0%
138	27,595,048	617	4,426	2,504	1,361	-3.8%
35	27,607,123	618	4,554	2,143	576	-3.8%
20	27,617,438	622	4,436	2,073	601	-3.8%
130	27,624,124	622	3,658	2,321	1,762	-3.7%
45	27,665,080	619	4,414	2,404	1,127	-3.6%
44	27,671,155	623	4,351	1,677	2,052	-3.6%

Continued on the next page

Table 11 – continued from previous page

<b>Relaxed RP</b>	<b>Obj. Function</b>	<b>Sol. Time</b>	<b>Dredge Time</b>	<b>Travel Time</b>	<b>Idle Time</b>	<b>Improv.</b>
11	27,701,164	616	4,079	2,235	2,512	-3.5%
133	27,704,531	619	4,008	2,500	1,769	-3.5%
30	27,740,583	617	4,340	2,357	1,754	-3.3%
82	27,781,756	621	4,166	2,482	1,392	-3.2%
57	27,845,139	621	4,085	2,893	538	-3.0%
135	27,927,844	617	4,713	2,221	1,284	-2.7%
110	27,994,967	615	4,480	2,293	585	-2.4%
52	28,025,874	614	3,910	2,508	1,283	-2.3%
75	28,094,342	615	4,390	2,807	1,337	-2.1%
89	28,123,443	621	3,866	1,642	1,291	-2.0%
6	28,150,651	617	4,690	2,713	1,655	-1.9%
40	28,197,995	623	4,669	2,520	1,361	-1.7%
88	28,285,445	619	3,804	1,761	1,143	-1.4%
120	28,296,058	612	4,062	1,973	844	-1.4%
112	28,316,172	612	4,450	2,208	1,580	-1.3%
9	28,317,045	619	4,747	1,956	1,406	-1.3%
27	28,373,795	623	4,403	2,215	372	-1.1%
50	28,399,629	619	4,225	1,953	795	-1.0%
126	28,461,538	618	3,780	2,676	1,147	-0.8%
107	28,488,247	616	4,018	2,052	1,665	-0.7%
7	28,489,816	621	4,318	2,298	1,352	-0.7%
24	28,500,806	620	4,117	2,458	2,177	-0.7%
79	28,529,994	620	3,908	1,980	2,036	-0.6%
53	28,548,495	611	3,924	2,149	1,464	-0.5%
87	28,553,147	620	4,840	2,078	887	-0.5%
18	28,606,224	619	3,997	2,691	1,224	-0.3%
31	28,607,658	616	4,646	2,896	1,191	-0.3%
47	28,611,624	620	4,087	2,533	972	-0.3%
43	28,676,714	616	4,576	2,803	1,552	-0.1%
0	28,697,012	616	4,337	2,815	1,727	0.0%
91	28,697,012	615	4,337	2,815	1,727	0.0%
92	28,697,012	617	4,337	2,815	1,727	0.0%
103	28,697,012	616	4,337	2,815	1,727	0.0%
10	28,697,143	615	4,340	2,812	1,731	0.0%
12	28,697,143	615	4,340	2,812	1,731	0.0%

Continued on the next page

Table 11 – continued from previous page

<b>Relaxed RP</b>	<b>Obj. Function</b>	<b>Sol. Time</b>	<b>Dredge Time</b>	<b>Travel Time</b>	<b>Idle Time</b>	<b>Improv.</b>
21	28,697,143	615	4,340	2,812	1,731	0.0%
46	28,697,143	613	4,340	2,812	1,731	0.0%
78	28,697,143	615	4,340	2,812	1,731	0.0%
81	28,697,143	618	4,340	2,812	1,731	0.0%
90	28,697,143	616	4,340	2,812	1,731	0.0%
93	28,697,143	618	4,340	2,812	1,731	0.0%
94	28,697,143	616	4,340	2,812	1,731	0.0%
95	28,697,143	618	4,340	2,812	1,731	0.0%
96	28,697,143	616	4,340	2,812	1,731	0.0%
97	28,697,143	617	4,340	2,812	1,731	0.0%
98	28,697,143	617	4,340	2,812	1,731	0.0%
99	28,697,143	617	4,340	2,812	1,731	0.0%
102	28,697,143	617	4,340	2,812	1,731	0.0%
104	28,697,143	614	4,340	2,812	1,731	0.0%
105	28,697,143	617	4,340	2,812	1,731	0.0%
129	28,697,143	617	4,340	2,812	1,731	0.0%
131	28,697,143	616	4,340	2,812	1,731	0.0%
132	28,697,143	616	4,340	2,812	1,731	0.0%
2	28,723,223	618	4,414	2,372	1,273	0.1%
63	28,724,368	617	3,969	2,435	1,452	0.1%
85	28,737,102	620	4,288	2,673	500	0.1%
14	28,860,349	620	4,314	2,205	1,288	0.6%
115	28,871,322	618	3,653	2,303	1,839	0.6%
33	28,877,678	617	4,585	2,351	1,768	0.6%
34	28,882,482	615	4,574	2,384	1,159	0.6%
16	28,915,927	616	4,120	2,281	1,719	0.8%
55	29,014,487	619	3,988	2,005	1,661	1.1%
32	29,083,974	618	4,452	2,694	1,394	1.3%
113	29,185,893	614	4,722	2,427	1,092	1.7%
80	29,272,317	616	4,305	2,731	1,997	2.0%
106	29,289,985	617	4,353	2,759	1,793	2.1%
64	29,334,588	620	4,287	2,481	1,356	2.2%
25	29,382,275	616	4,333	2,597	1,405	2.4%
100	29,469,109	622	4,798	2,985	1,421	2.7%
101	29,469,109	618	4,798	2,985	1,421	2.7%

Continued on the next page

Table 11 – continued from previous page

<b>Relaxed RP</b>	<b>Obj. Function</b>	<b>Sol. Time</b>	<b>Dredge Time</b>	<b>Travel Time</b>	<b>Idle Time</b>	<b>Improv.</b>
76	29,491,291	613	4,220	2,249	1,797	2.8%
42	29,658,625	615	4,506	2,399	1,287	3.4%
4	29,691,787	610	4,589	2,940	1,534	3.5%
58	29,777,085	618	4,874	2,180	1,231	3.8%
137	29,902,776	616	4,434	2,430	1,424	4.2%
127	30,239,060	619	4,588	2,345	2,174	5.4%
17	30,296,936	614	3,961	2,200	1,509	5.6%
19	32,439,534	620	4,866	1,820	1,741	13.0%

As shown in the last row of Table 11, the restricted period number 19 has the largest improvement in objective function (13%) with 3,742,522 cubic yards of dredging. This period is associated with the dredging job CHANNEL ISLANDS HARBOR, CA with the range of [91, 258].

Table 12 shows the impacts of all restricted periods of dredging jobs by allowing them to be dredged at different percent of relaxation from 0% to 50% with 25% increment. Similar to Section 3.2.1, in implementing the problem instances, the budget limitation has been removed from the model to focus on the impacts of restricted periods. As mentioned in Section 3.2, the test problems run on a Core(TM) i7 CPU @ 2.93 GHz, 8 GB RAM computer.

Table 12: All Restricted Periods Relaxation Impacts in MCP-DS model (objective function: cubic yards dredging, duration: days)

<b>Relax.</b>	<b>Obj. Function</b>	<b>Sol. Time</b>	<b>Dredge Time</b>	<b>Travel Time</b>	<b>Idle Time</b>	<b>Improv.</b>
0	10,890,124	14,531.4	1,114	2,832	1625	0.0%
25	17,929,568	14,541.2	1,923	752	0	65%
50	19,588,573	14,573.6	2,308	712	0	80%

According to Table 12, we could improve the objective function of total cubic yards of dredging up to 65% and 80% by allowing all restricted periods to be delegable at 25% and 50%, respectively. In this model by allowing 50% dredging in all restricted periods we can dredge 19,588,573 cubic yards out of 21,191,892 cubic yards of all jobs. This is 92% of all possible amount of dredging versus 51% in the model without dredging in the restricted periods (the first row of the Table 12).

## 5 Recommendations and Conclusions

This work has offered a highly generalized dredge scheduling optimization framework for use by dredge planners. The work has already been transferred to USACE computing systems and various versions of the developed model have been utilized in support of planning efforts on the West and East coast. The results of the project show that partial dredging, dredge maintenance, modified mob/demob costs/budgets, operations rates, multiple dredges per job and multiple visits to jobs can all be allowed for in a constraint programming platform. Using this platform, feasible solutions can be obtained to this complex model in a matter of minutes or hours. Evaluating the potential benefit on cubic yards dredged by considering each model enhancement suggests that these new flexibilities are significant for guiding practitioners to solutions. That is, adding the discussed flexibilities to the models make a significant difference in the solutions obtained.

With a more flexible model and the increased potential for significant cubic yards dredged gains comes a new set of computational challenges. In addition to revealing how to model additional problem features, this project has revealed a number of new methodological challenges that need to be explored. That is, with increased flexibility comes a much larger solution space for any optimization methodology to explore. While one solution to this problem is to use the solution from a simplified model as a seed solution to the more complex model, more sophisticated approaches are certainly worthy of exploration. In the course of studying these issues, the investigators note that many aspects of the expanded problem formulation (e.g. schedule of an individual dredge) decompose nicely. That is, there are components of the scheduling problem that can be thought of in separate pieces. The acknowledgement of this fact leads the investigators to believe that opportunities to implement the existing constraint programming approach in a parallel computing system could yield immediate solution improvements. Moreover, the complexities of the new problem suggest that it is now appropriate to formally study the parameters utilized in the constraint programming search. While these values were not of significance in the base model, the newly identified computational challenges mean that implementation details are now far more significant.

## References

- [1] Dena D Dickerson, Kevin J Reine, and Douglas G Clarke. Economic impacts of environmental windows associated with dredging operations. Technical report, DTIC Document, 1998.
- [2] Ridvan Gedik, Chase Rainwater, Heather Nachtmann, and Ed A Pohl. Analysis of a parallel machine scheduling problem with sequence dependent setup times and job availability intervals. *European Journal of Operational Research*, 251(2):640–650, 2016.
- [3] IBM. Ibm ilog cplex optimization studio cp optimizer users manual version 12 release 6. [http://www-01.ibm.com/support/knowledgecenter/SSSA5P\\_12.6.1/ilog.odms.studio.help/pdf/usrcoptimizer.pdf](http://www-01.ibm.com/support/knowledgecenter/SSSA5P_12.6.1/ilog.odms.studio.help/pdf/usrcoptimizer.pdf). Accessed: January 2015.
- [4] IBM. Ibm ilog cplex optimization studio v12.3, 2011. <http://pic.dhe.ibm.com/infocenter/cosinfoc/v12r3/index.jsp>. Accessed: October 2013.
- [5] Harjunkoski Iiro and Grossmann Ignacio E. Decomposition techniques for multistage scheduling problems using mixed-integer and constraint programming methods. *Chem. Engng*, (26):1533–1552, 2002.
- [6] Hooker John N. Integrated methods for optimization (international series in operations research and management science). *Springer-Verlag New York, Inc., Secaucus, NJ, USA*, 2006.
- [7] Lombardi Michele and Milano Michela. Optimal methods for resource allocation and scheduling: a cross-disciplinary survey. *Constraints*, (17):51–85, 2012.
- [8] Heather Nachtmann, Kenneth Mitchell, Chase Rainwater, Ridvan Gedik, and Edward Pohl. Optimal dredge fleet scheduling within environmental work windows. *Transportation Research Record: Journal of the Transportation Research Board*, (2426):11–19, 2014.
- [9] Heather Nachtmann, Edward Pohl, Chase Rainwater, and Ridvan Gedik. Resource allocation for dredge scheduling and procurement: A mathematical modeling approach. *The Report submitted to Coastal and Hydraulics Lab U.S. Army Engineer Research and Development Center*, October 2013.
- [10] U.S. Army Corps of Engineers. Inland waterway navigation: Value to the nation. <http://www.corpsresults.us/>. Accessed: July 2015.

- [11] Topaloglu Seyda and Ozkarahan Irem. A constraint programming-based solution approach for medical resident scheduling problems. *Computers and Operations Research*, (38(1)):246–255, 2011.
- [12] Jain Vipul and Grossmann Ignacio E. Algorithms for hybrid milp/cp models for a class of optimization problems. *INFORMS Journal on Computing*, (13):258–276, 2001.
- [13] van Hoeve Willem-Jan and Katriel Irit. *Handbook of Constraint Programming*. Elsevier, 2006.

# Appendix

Table A1: 116 Project Properties (volumes: CY, costs: USD)

Job ID	Volume	Cost	Job ID	Volume	Cost
000030	439,726	3,201,839	011810	577,711	2,972,600
000360	900,709	5,533,068	011860	156,607	1,104,938
046063	4,376	46,441	011880	30,523	420,827
074955	2,267,192	14,477,345	012030	544,338	2,338,424
000950	466,950	2,989,574	012550	123,064	9,739,760
001120	2,001,129	2,523,736	008190	174,603	998,309
088910	39,308	1,016,772	072742	26,937	644,784
010222	178,088	791,822	012801	67,578	318,000
076060	451,796	1,261,920	012990	217,888	967,081
080546	6,723	275,719	073567	34,637	302,055
002080	2,472,603	6,685,844	013080	723,937	2,628,970
002250	102,032	1,242,273	013330	44,401	334,654
041015	85,093	2,409,673	013590	119,668	1,891,959
003630	277,836	786,758	013680	1,193,406	2,009,923
002440	2,890,491	3,793,482	013880	252,670	251,296
002410	179,782	1,612,871	013940	192,277	980,108
002620	116,357	2,307,509	014310	82,949	748,816
002640	396,079	909,977	076031	46,686	481,990
014360	5,413,965	5,452,500	014370	4,510	102,371
008160	67,221	1,231,600	021530	26,009	144,042
003130	13,252	226,709	014760	59,003	690,963
076106	35,672	321,356	015100	572,395	2,405,442
022140	45,533	142,900	015280	95,491	723,544
003600	808,778	1,502,833	015600	21,003	178,236
003840	397,516	1,745,287	087072	83,378	146,508
004550	243,898	1,489,330	087455	32,688	453,483
004610	38,598	306,499	015870	295,967	1,881,768
004710	201,116	1,122,792	057420	231,639	1,709,816
004800	117,090	719,437	016130	833,305	2,509,084
005050	80,528	733,469	076063	120,808	900,546
005220	191,015	1,708,370	074709	145,537	942,239
005700	261,440	1,058,165	016550	261,985	1,363,696
005880	1,117,205	9,124,564	067318	127,064	310,965

Continued on the next page

Table A1 – continued from previous page

<b>Job ID</b>	<b>Volume</b>	<b>Cost</b>	<b>Job ID</b>	<b>Volume</b>	<b>Cost</b>
041016	63,380	2,260,932	073644	572,249	4,008,166
006260	186,551	1,183,650	016800	216,709	864,890
006480	668,425	2,073,745	016860	47,674	284,901
006670	41,563	311,454	017180	22,153	159,881
006770	577,424	1,543,516	017370	306,546	5,944,930
006910	147,811	2,153,095	074390	633,833	8,574,738
007150	1,038,304	1,534,705	017300	64,118	1,162,671
007610	42,408	283,559	017350	42,577	389,861
007810	167,704	1,416,099	017380	49,558	2,497,492
007860	1,494,596	4,048,374	017760	64,262	950,325
008410	1,189,684	12,991,774	017720	212,214	1,588,367
054000	225,664	1,427,334	017960	1,037,987	4,895,841
008430	283,367	1,151,256	073598	229,090	456,000
010020	67,571	380,810	018710	55,762	326,262
010040	80,000	1,579,250	018750	105,955	443,959
074719	122,930	864,000	024190	1,086,812	1,486,174
010310	102,424	751,304	019550	97,935	442,630
010490	74,288	519,202	019560	50,777	331,749
010580	261,769	1,845,812	039023	9,868	66,150
011060	59,190	419,900	019990	53,971	258,289
011270	40,729	530,127	020040	323,758	1,262,279
000068	681,961	1,419,778	020030	1,171,297	6,527,537
011410	944,417	1,496,737	072852	33,939	4,687,087
000063	1,505,100	5,388,149	020290	75,373	468,695
011670	1,282,956	2,509,501	073803	561,192	2,499,452
			<b>Total:</b>	48,305,584	223,012,020

Table A2: Production Rates of Dredge Vessels (cubic yards/day)

Row	Vessel	Rate
1	BARNEGAT BAY DREDGING COMPANY	1,238
2	PORTABLE HYDRAULIC DREDGING	1,301
3	TNT DREDGING INC	1,637
4	ROEN SALVAGE COMPANY	1,962
5	LUEDTKE ENGINEERING CO.	1,989
6	MADISON COAL & SUPPLY CO.	2,296
7	CURRITUCK	2,375
8	M.C.M. MARINE INC.	2,709
9	SOUTHWIND CONSTRUCTION CORP	2,855
10	LAKE MICHIGAN CONTRACTORS, INC	3,311
11	KING COMPANY, INC.	3,481
12	COTTRELL ENGINEERING CORP.	3,728
13	FRY	3,941
14	MERRITT	4,532
15	GOETZ	5,941
16	B+B DREDGING CORPORATION	6,837
17	WRIGHT DREDGING CO.	6,965
18	MARINEX CONSTRUCTION CO INC	8,332
19	SOUTHERN DREDGING CO., INC.	8,443
20	YAQUINA	9,007
21	WEEKS MARINE, INC (ATLANTIC)	10,436
22	LUHR BROS. INC.	10,478
23	MCFARLAND	10,959
24	KING FISHER MARINE SERV., INC.	12,347
25	NORFOLK DREDGING COMPANY	12,882
26	NATCO LIMITED PARTNERSHIP	15,556
27	GULF COAST TRAILING CO.	17,080
28	GREAT LAKES DREDGE & DOCK CO.	17,282
29	MIKE HOOKS INC.	17,537
30	PINE BLUFF SAND & GRAVEL CO.	19,245
31	MANSON CONSTRUCTION CO	21,726
32	HURLEY	24,618
33	WEEKS MARINE, INC.(GULF)	29,147

Continued on the next page

Table A2 – continued from previous page

<b>Row</b>	<b>Vessel</b>	<b>Rate</b>
34	POTTER	32,841
35	ESSAYONS	33,870
36	BEAN STUYVESANT, LLC	34,716
37	T.L. JAMES & CO., INC.	35,324
38	BEAN HORIZON CORPORATION	38,665
39	WHEELER	41,463
40	JADWIN	66,418