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Measurement of Traffic Network Vulnerability for Mississippi Coastal Region

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Abstract

Natural disasters such as a hurricane can cause great damages to the transportation networks and significantly affect the evacuation trip operations. An accurate understanding and measurement of the network vulnerability can enhance the evacuees' preparedness and responding capabilities during an emergency incident. This study presents a game theory based approach to the analysis of the network vulnerability under a hurricane evacuation. A game is constructed between a router, who is committed to seek the minimum-cost path for the evacuation travelers, and a tester, who wants to maximize the travel cost by disturbing the links. In the game process, the distribution of evacuation demand is elastic because the probability of selecting an evacuation destination is determined by the path risk and travel cost. In addition, the congestion effect is considered, and a solution strategy based on the method of successive averages (MSA) is adopted. Over a sample network, the proposed method is compared with other three methods for the network vulnerability analyses. Furthermore, the method is applied to the vulnerability analysis of a large scale network in Mississippi Gulf coast area.

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1 Project Description

The measurement of transportation network vulnerability captures the network weaknesses or susceptibility to threats affected operational performance, and plays an important role in transportation networks analysis (1, 2). Understanding the specific significance of transportation network vulnerability can improve the capability of a transportation agency when dealing with the impacts of interrupting threats in network planning, design, and management. In addition, it is useful to enhance the traveler's ability to respond to disastrous events and emergency incidents (3). As a result, various studies targeting on vulnerability assessment under conventional and disastrous conditions were conducted in recent years (2, 3, 4, 5). Typically in such a study, the origin-destination travel demand is assumed to be known, and the vulnerability assessment results are mainly dependent on link travel time, networks topological structure, and the adopted measurement methods.

The presence of a disastrous condition plays an important role in the modeling of trip distribution and then the vulnerability measurement. In the traffic assignment at a conventional traffic condition, the trip demands can be allocated to destinations proportionally to the populations of the possible destinations. In contrast, under an emergency evacuation situation, the evacuation trip demand would be allocated after an aggregating analysis of the travel distance, and link risk, in addition to the consideration of the sheltering and handling capacities at the destinations (6). Specifically, evacuees make decisions on trip destinations based on the assessment of the risk and cost, which means that the modeling methods of travel demand and trip distribution between a conventional traffic operation and an emergency evacuation are quite different. This research study will address these differences in vulnerability measurement methodology by introducing an evacuation destination selection mechanism.

Game-theory based risk modeling methods have been adopted recently in network vulnerability studies (7, 8, 9, 10). In the game theoretic model a game is played between a benevolent router and malevolent network tester to find link failure probabilities. The link failure probabilities are used to indicate the network vulnerability and the links with the highest failure probabilities are the most critical links in the network. The information on the link failure probabilities can help the travelers determine trip and route strategies accordingly and achieve a more reliable system-level outcome (9, 10). Therefore, the primary objective of this study is to develop a game-theoretic approach to the analysis and measurement of networks vulnerability under hurricane evacuation.

A lot of research efforts in the measurement of vulnerability performance of a transportation network have been conducted. Berdica had a comprehensive literature review and investigation on how the road vulnerability related problems were addressed in the past, and what the solutions to the problems should be in nowadays and for the future (1). However, this study only reviewed the vulnerability research at a qualitative level and the proposed solution strategies remained in conceptualization. As to quantitative approaches, numerous studies were undertaken extensively for the measurement of network vulnerability (2, 3, 5, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26).

The methods proposed in these studies generally fall into the following three types. Firstly, the risk measurement models were built based on network topology. When random incidents or failures occur on the network, the topology indexes would vary generating representing information that leads to the estimates of link/network vulnerability. Under this type of modeling, various estimate models including the networks minimum cuts or mincuts, link importance, and link using rate were introduced to reflect the level of vulnerability of a network. Tu et al. used the networks mincuts (11) and Jenelius et al. derived the link importance to measure vulnerability respectively (2). Hu et al. tested the urban road networks in four cities using the network topology analysis (15), and Han et al. designed a variety of simulation scenarios for network interruption to assess network vulnerability (16). Secondly, there are quite a few vulnerability studies based on networks accessibility, especially using the accessibility index initially developed by Hansen (20). Typically, accessibility refers to the ease of reaching opportunities for activities and services and can be used to assess the performance of a transportation network. Chen et al. used network-based accessibility measures to assess vulnerability of degradable transportation networks. The network-based accessibility measures quantified the consequence of one or more link failures in terms of network travel time or travel cost increase plus the effect of behavioral responses of users due to the failure in the network (18). Thirdly, the game theoretic approach has been successfully applied to network vulnerability studies recently. The method hypothesizes a 'game' situation in which a router constantly seeks the lowest-cost route, and a tester has the power to fail a critical road link to cause the most expensive travel cost to the router (8). Bell et al reviewed the application, mathematical formulation and solution algorithm of game model in road vulnerability (25). Generally a mixed optimization process is used in this method. Link-use probabilities is optimal for the router, and link-failure probabilities is optimal for the tester. Finding the equilibrium involved solving a maxi-min programming problem. When link costs are fixed (not traffic-dependent), the maxi-min problem can be recast as a linear programming problem. Where link costs are traffic-dependent (e.g., where queuing is a feature), the mixed strategy Nash equilibrium can be found by a numerical method of successive averages. To model the different characteristics of travel behavioral responses, a combined travel demand model is needed to estimate the long-term equilibrium network condition due to network disruptions.

It would be obvious to state that the research methods based on network topology may only relate vulnerability measurement to network connectivity and topology but fail to provide a framework of procedures considering the travelers' evacuation behavior and responses to interrupted network links or nodes under an emergency evacuation condition. In contrast, the methods using network accessibility consider both the consequence of a network failure in terms of increased travel cost and travel time and the effect of travelers' behaviors and responses to an emergency situation. Therefore the network vulnerability analysis using the accessibility modeling method could be applied to a disastrous evacuation condition. Furthermore, compared with the accessibility modeling method, the game theoretic method not only well reflects the traveler's behavior of constantly seeking the lowest travel cost/time in a 'shortest path' but it also captures the nature of the problem of identifying the critical links and quantifying the vulnerability of the network. The mixed optimization process nicely includes the seeking of both the shortest paths and the critical links in a 'game' problem. This study contributes to the emergency management research area by introducing the game theoretic method to the network vulnerability analysis under an emergency evacuation condition, due to an assumed hurricane disaster in the Gulf coast area.

There are two major questions that need to be addressed for a network at the Gulf coast under hurricane evacuation condition. The first question is how to describe the link risk, and the second one is how to predict the evacuation trip demand and process the trip distribution (6). After the disastrous 2005 hurricane Katrina, both the Federal Emergency Management Agency (FEMA) and the US Army Corps of Engineers (USACE) undertook intensive efforts to update coastal hazard information using specially developed methods, in which probabilities were used to present the link risk. Neidoroda et al. developed the flood elevation-frequency curves for a dense network of points throughout the Mississippi Gulf coast area, suggesting that the flood peak surge heights follow the Gauss distribution (27). Sohn also utilized the flood probability of a road link to represent the link risk, and conducted an analysis to assess the vulnerability of highway network links in Maryland in case of flood damage (22).

Pel et al. reviewed the trip decisions on how and where the hurricane affected populations were evacuated and suggested to reveal the major decision factors by using both stated preferences in a survey and real observed data (6). Cheng developed a study to calibrate the friction factors for hurricane evacuation trip distribution. In the study the observed origin-destination matrix was reconstructed based on a survey data and trip distribution models were estimated to produce the best fitting to the origin-destination matrix, and the lengths of the evacuation trips showed statistical regularity (28). Therefore, in a later study the same data set was used to estimate two multinomial logit models (29). It was found that, as expected though, the parameters for travel cost and the probability that the destination choice was at risk by hurricane were negative, indicating that the destination with a larger cost and a higher risk is less likely to be chosen. A more significant contribution of Cheng's research would be the proposed destination choice model which was used to present evacuation behaviors. With the destination choice model, the probability of choosing the destination from the evacuation area can be calculated by inputting the known values for the dependent variables. On the other hand, the destination choice model also provides an approach to searching for the evacuation routes and therefore the evacuation trip demands for the chosen routes can be calculated through multiplying the choosing probabilities with the total evacuation traffic generation in a traffic analysis zone (TAZ).

In almost all of the previous game theoretic models used for network vulnerability analyses, the route use rates are defined as the ratios of the traffic demands in the shortest paths to the total traffic demand (30), which are neither elastic nor affected by the risk of the incident such as a hurricane. In this study, a new game theoretic formulation with elastic constraint for network vulnerability is developed. Compared with previous studies, three newly elements have been adopted in the study. Firstly, drivers no longer make their route choice solely considering their own utility but rather based on the network 'dispatcher'. Secondly, link risk and travel cost affect the route decision probability. Thirdly, the Bureau of Public Roads (BPR) function is used to consider the effect of traffic volume on the vulnerability of the network.

The remainder of this project is organized as follows: In the following section game theoretic model is proposed, and the method of successive averages (MSA) is applied to solve the problem. Section 3 presents our sample network and summarizes preliminary computational results used to testify model performance. Based on these results, we apply the model and solution method to a realistic large-scale evacuation network in Section 4, and we discuss benefits obtained via applying our modeling and solution approach. Conclusions are drawn in Section 5.

2 Methodological Approach

In this project, it is assumed that there are two opponents in a non-cooperative game with symmetric information: a router, who seeks the least-cost path to the chosen evacuation destination and assigns the evacuation demand in the path according to the choice probability, and an evil tester, who strives to maximize the trip cost to the router. The mixed strategies are adopted, which means that the use or failure of the network is determined by the shortest paths or the worst scenario probabilities. Elastic demand is assumed and traffic congestion effect is incorporated.

2.1 Game-Theoretic Model

In order to describe the game model, the main decision variables are designed to have two parts that are the vector of link choice probabilities $\mathbf{P}$, and the vector of link failure probabilities $\mathbf{q}$. The notations used in the formulation of the problem are summarized in Table 1.

Notations

Table 1 Summary of Notations

$\mathbf{P}$	Vector of link choice probabilities; p_i is probability of link *i* to be chosen by the network router
$\mathbf{q}$	Vector of link failure probabilities; q_j is probability of link *j* to be disturbed by the network tester
$i \in \mathrm{E}$	Link *i,* which belongs to set of links *E*
$j \in \mathrm{E}$	Scenario *j*, which denotes link *j* is disturbed
$s \in \mathrm{S}$	Evacuation origin node *s*, which belongs to set of origin nodes *S*
$v \in \mathrm{V}$	Evacuation destination node v, which belongs to set of destination nodes *V*
$k \in \mathrm{K}$	Shortest path *k* from *S* to *V*, which belongs to set of paths *K*
TC	Total travel cost of network; tc_i is the expected travel cost of link *i*
$flow_i$	Traffic flow on link i
h_k	Traffic flow on path k
t_i^0	Free flow travel cost on link i at initial computation iteration
$t_{i,j}$	Travel cost on link i under scenario j
d_k	Travel cost on path k
f_i	Risk on link i due to flooding
r_k	Risk on path k
o_s	Generation of trip demand on origin node s
$a_{i,k}$	Parameter that takes value 1 if link i is on path k, 0 otherwise
$b_{s,k}$	Parameter that takes value 1 if path k starts at node s, 0 otherwise
$c_{v,k}$	Parameter that takes value 1 if path k ends at node v, 0 otherwise
α	The parameter in BPR function, which is 0.15 after reference
β_0	The parameter in BPR function, which is 4.0 after reference
β_1	Impact factor for travel cost
β_2	Impact factor for risk due to flooding
θ	Degree of selectiveness for the tester to disturb specific links
DC	Disruption cost factor
ε	Convergence criterion for computation

The notation scenario *j* means that link *j* is disturbed, and the other links are in normal use. Under any scenario, the state of each link is either disturbed or not disturbed. If link *i* is not disturbed, it remains with its original travel cost t_i^0; otherwise when the link is disturbed, the link's travel cost will increase to a much higher level by multiplying a disruption cost factor (DC), which is considered to be a big constant value. The evacuation destination nodes, and evacuation origin nodes are limited and known so that the evacuation paths can be recalculated when a particular link is disturbed. As a result, the change of the assignment of the evacuation traffic demand may take place. In the traffic assignment process, the increase of travel cost and the encountered flooding risk have negative effects on the evacuation route and destination choices. When each player has no more incentive to move a different strategy, the game will end. In the process for an equilibrium to be achieved, on one hand, the higher link use probability means a safer path choice, on the other hand, the higher link failure rate means that the disturbing of this link leads to more total travel cost loss and is more critical in the network, which actually also represents the network vulnerability.

Formulation of Problem

$$\min_{\mathrm{p}}\max_{\mathrm{q}}\mathrm{TC}(\mathrm{q},\mathrm{p}) = \sum_{i\in E} p_i \cdot tc_i = \sum_{j\in E}\sum_{i\in E} q_j p_i t_{i,j} \tag{1}$$

Subject to:

$$\sum_{j\in E} q_j = 1; \quad q_j \ge 0 \qquad \forall j \in E \tag{2}$$

$$p_i = \frac{\sum_{k\in K} h_k a_{i,k}}{\sum_{k\in K} h_k} \qquad \forall i \in E \tag{3}$$

$$r_k = \max_{i\in E}\left(f_i \cdot a_{i,k}\right) \qquad \forall k \in K \tag{4}$$

$$d_k = \sum_{i\in E}\left(tc_i \cdot a_{i,k}\right) \qquad \forall k \in K \tag{4'}$$

$$h_k = \frac{e^{(\beta_1 d_k + \beta_2 r_k)} b_{s,k}}{\sum_{x \in K} \left(e^{(\beta_1 d_x + \beta_2 r_x)} b_{s,x} \right)} o_s \qquad \forall k \in K \tag{5}$$

The game of the two players is formulated as a minimax problem presented in Equation 1. Equation 2 is the constraint condition for the failure probabilities, which are between 0 and 1, and the summation of the probabilities equals 1. Equation 3 defines the link choice probability that is the ratio of the sum of demands of all paths using the link to the total demand. According to the bottleneck theory, Equation 4 states that the maximum link risk encountered by the path is defined as the risk of the path. Similarly, Equation 4' calculates the travel cost of the path. Equation 5 is used to depict how the evacuation demand is assigned to the each path from an evacuation origin, where parameter β_1 is impact factor for the travel cost through a path that connects the origin node with the destination node, and the parameter β_2 is impact factor for the path risk under hurricane evacuation through the path connecting the origin and the destination nodes. Following Cheng's study in 2008, the two parameters β_1 and β_2 are set at -0.05 and -0.5 respectively (29).

2.2 Solution Methodology

The model in Equations 1 through 5 is a minimax problem, which is an NP-hard problem. Sheffi in 1985 used the method of successive averages (MSA) to solve such kind of problems (31), in which the players make decisions based on the history of the opponent's strategies. Bell found that the minimax problem with game theory can be formulated as a linear programming problem (7). In addition, some previous experiences showed that using the MSA strategy may obtain an approximate solution to this problem. Qiao provided a formal proof for the convergence of the MSA solution method, while the BPR function was introduced and used to describe a congestion effect (10). In this study, Equation 3 indicates that the inequation $0 \leq \mathrm{p_i} \leq 1$ is a true statement. In addition, the objective function is only related to three variables ($\mathrm{p_i}$, $\mathrm{q_j}$, and $\mathrm{t_{i,j}}$), and the variable $\mathrm{t_{i,j}}$ is related to the other two variables ($\mathrm{p_i}$, $\mathrm{q_j}$). The above two facts make the problem model in Equations 1 through 5 meet the same formats in Bell's model and Qiao's model (please refer to Qiao's paper of 2014 for detail proof of convergence of solution to these problems). As a result, the solution strategy of MSA is also applicable and used to solve the model of this problem. The following are the algorithm procedures followed to find the solution.

Step 0: Initialize, $\mathrm{q}_\mathrm{j}^0 = 1/\mathrm{L}$, $\mathrm{p}_\mathrm{i}^0 = 0$ and n (number of iterations) = 1, where L is the number of network links.

Step 1: Calculate tc_i^n, the expected travel cost of link i, as shown in Equation 6.

$$tc_i^n = \sum_{j \in E} t_{i,j}^n q_j^n \tag{6}$$

Step 2: Calculate the shortest evacuation paths from S to V. Using the $\mathrm{tc}_\mathrm{i}^\mathrm{n}$ calculated, the Floyd-Warshall algorithm (32) is used to identify the shortest paths and determine the dummy variables of $\mathrm{a}_{\mathrm{i,k}}$, $\mathrm{b}_{\mathrm{s,k}}$, and $\mathrm{c}_{\mathrm{v,k}}$. Then update the path risks and costs using Equation 4, calculate the choice probability of each path in $\dfrac{e^{\beta_1 d_k + \beta_2 r_k} b_{s,k}}{\sum_{x \in K} e^{\beta_1 d_x + \beta_2 r_x} b_{s,x}}$, and assign the evacuation demand according to the probabilities of choice for the paths using Equation 5.

Step 3: Calculate the traffic flow on link i, using $flow_i^n = \sum_{k \in K} h_k^n a_{i,k}$. Update $\mathrm{t}_\mathrm{i}^\mathrm{n}$ by BPR function expressed in the following equation where cap_i is the capacity of link i.

$$t_i^n = t_i^0 \left[1 + \alpha \left(\frac{flow_i^n}{cap_i} \right)^{\beta_0} \right] \tag{7}$$

Step 4: Calculate auxiliary link use probability $\mathrm{y}_\mathrm{i}^\mathrm{n}$ using Equation 3. Update link use probability (MSA).

$$\mathrm{p}_i^n = \left(\frac{1}{n} \right) y_i^n + \left(1 - \frac{1}{n} \right) \mathrm{p}_i^{n-1} \tag{8}$$

Step 5: In this study, a logit function as shown in Equation 9 is adopted to calculate the link disturbance probability (within a varying degree), rather than seeking the worst $\mathbf{q}$ (8). In Equation 9, the parameter θ is used to represent the degree of selectiveness or aggressiveness to disturb the links. For any two links, there may be a difference in the extent to maximize the network total cost, a larger θ may lead to more disparity of disturbance probability between the two links. When θ is zero, the evil tester would be indiscriminate for all the links.

$$q_j^n = \frac{\exp\left(\theta \cdot \sum_{i \in E} p_i^n t_{i,j}^n\right)}{\sum_{e \in E} \exp\left(\theta \cdot \sum_{i \in E} p_i^n t_{i,e}^n\right)} \tag{9}$$

Step 6: Update travel cost $t_{i,j}^n$ on link i under scenario j, where the disruption cost factor (DC) is set at 10, which is considered a big number for the model.

$$t_{i,j}^n = \begin{cases} (DC) \cdot t_i^n & i = j \\ t_i^n & i \neq j \end{cases} \tag{10}$$

Step 7: Check termination criteria. Bell came up with a weighted entropy into the objective function (8). Equation 1 can be improved in the following equation.

$$\min_p \max_q \mathrm{TC}(q, p) = \sum_{j \in E} \sum_{i \in E} q_j p_i t_{i,j} + \left(\frac{1}{\theta} \sum_{j \in E} q_j \ln q_j\right) \tag{11}$$

When the game achieves equilibrium, the total cost will change weakly. In this study, if $|TC(q, p)^n - TC(q, p)^{n-1}| \leq \varepsilon$, then the computation stops, otherwise set n = n+1 and return to Step 1.

3 Results/Findings

3.1 Computation Results

The effectiveness of the proposed method and solution procedure are tested by a sample network. As shown in Figure 1 the sample network is designed to provide basic network components with seven nodes and twelve links. Each link is marked with a link number (letter), link free-flow travel cost, and link flooding risk. There are two evacuation destination nodes, two evacuation origin nodes with 1,000 trips in evacuation demand.

The solution algorithm is coded in Matlab and run in a Dell Precision M4800 laptop computer with i7 CPU at 2.9 GHz and with 16 GB memory. In the iterative process, the objective function achieves convergence in about 10 seconds with 200 iterations.

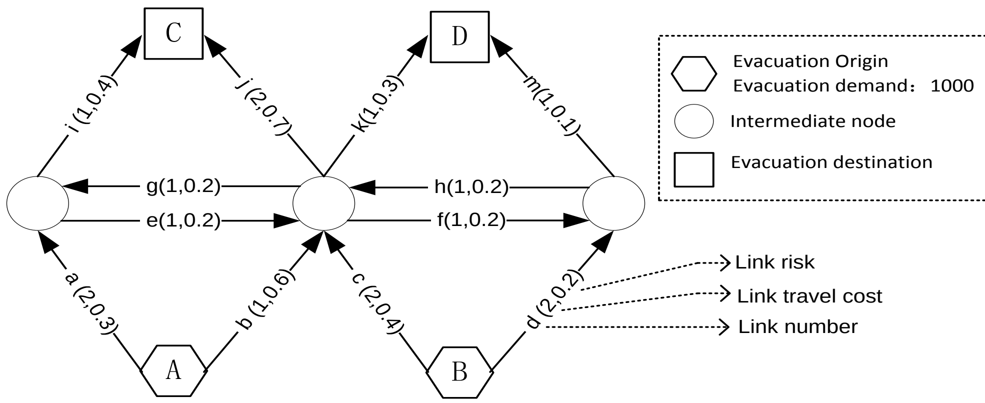


Figure 1 Sample test network

Table 2 presents the solution process for the example problem as it proceeds with the first two iterations. At the initial condition, the tester does not yet know how the travelers seek the evacuation paths, and therefore all link use probabilities and link failure probabilities are uniformly distributed, i.e., $q_j^0 = 1/L$, and $p_i^0 = 0$. Similarly, each link travel cost is equal to the link's free flow travel time. After the initial information is set, the travelers seek all the shortest paths from evacuation origins to evacuation destinations based on the expected link costs. By combining the path total cost and risk, the evacuation demand assignment h_k^1 is achieved. Then, link flow $flow_i^1$ is calculated, and link travel cost t_i^1 is updated by BPR function. Through Equation 3, the link use probability p_i^1 is calculated. Hence, the tester produces its strategy q_i^1 according to Equation 9.

In the second iteration, the computational procedure repeats as in the first iteration. It is worth noting that the shortest paths are identified based on the expected link cost tc_i^1 (which is updated with Equation 6) rather than the link travel cost t_i^1. The expected link cost is equal to the link travel cost t_i^0 in the first iteration because all the link failure probabilities are the same at the initial condition.

In contrast to the first iteration, the most remarkable shift is that link c and j are no longer used in the second iteration. The possible reason would be that the link failure probabilities of the two links are 89% and 4.1%, which are the top two in the first iteration. It means the network total cost will encounter the most increases if these two links are disturbed, which means the tester would love to disrupt the two links. Then, the expected cost in these links are increased so that the travelers would avoid them when choosing the evacuation routes.

TABLE 2 Computation Results of First and Second Iterations

1st Iteration								
Paths				Links				
Evacuation OD Pair	Shortest Path, (Expected Travel Cost, Travel Risk)	Probability of Choice	h_k^1	Link ID	$flow_i^1$	t_i^1	p_i^1	q_i^1
A-C	b-j, (3, 0.7)	0.2316	232	a	269	2.001	0.135	0.006
	a-i, (3, 0.4)	0.2691	269	b	731	1.021	0.366	0.019
	b-g-i, (3, 0.6)	0.2434	243	c	725	2.040	0.363	0.890
A-D	b-k, (2, 0.6)	0.2559	256	d	275	2.001	0.138	0.007
B-C	c-j, (4, 0.7)	0.2143	214	e	0	1.000	0.000	0.000
	c-g-i, (4, 0.4)	0.2489	249	f	0	1.000	0.000	0.000
B-D	c-k, (3, 0.4)	0.2617	262	g	492	1.004	0.246	0.005
	d-m, (3, 0.2)	0.2751	275	h	0	1.000	0.000	0.000
				i	761	1.024	0.381	0.023
				j	446	2.006	0.223	0.041
				k	518	1.005	0.259	0.006
				m	275	1.000	0.138	0.002
2nd Iteration								
Paths				Links				
Evacuation OD Pair	Shortest Path, (Expected Travel Cost, Travel Risk)	Probability of Choice	h_k^2	Link ID	$flow_i^2$	t_i^2	p_i^2	q_i^2
A-C	a-i, (3.329, 0.4)	0.5113	511	a	511	2.010	0.256	0.246
				b	489	1.004	0.244	0.015
				c	0	2.000	0	0.097
A-D	b-k, (2.232, 0.6)	0.4887	489	d	1000	2.145	0.500	0.487
B-C	d-h-g-i, (5.396, 0.4)	0.4473	447	e	0	1.000	0	0.001
				f	0	1.000	0	0.001
B-D	d-m, (3.166, 0.2)	0.5527	553	g	447	1.003	0.224	0.001
				h	447	1.003	0.224	0.001
				i	958	1.061	0.479	0.017
				j	0	2.000	0	0.097
				k	489	1.004	0.244	0.015
				m	553	1.007	0.277	0.023

3.2 Comparison of Game Models

Three previous models that utilize game theory to measure network vulnerability are also implemented for the sample network, and the results are compared with ours.

Table 3 Comparison of Results of Four Models†

Model Developed						Qiao's Model			Bell's Model			Lownes' Model		
Result 1 (β_2 =-0.5)			Result 2 (β_2 = 0)			(2015)			(2008)			(2011)		
ID	Failure (%)	Use (%)	ID	Failure (%)	Use (%)	ID	Failure (%)	Use (%)	ID	Failure (%)	Use (%)	ID	Failure (%)	Use (%)
d	41.8	25.0	d	42.7	25.0	c	50.0	25.0	c	50.1	25.1	c	21.5	25.1
c	28.6	25.0	c	29.2	25.0	d	49.9	25.0	d	49.8	24.9	d	20.8	24.9
b	6.5	33.5	b	6.7	33.5	j	0.1	0.1	j	0.1	0.1	b	10.1	35.2
j	6.3	16.8	a	4.9	16.5	b		25.0	b		25.0	i	10.1	35.2
a	4.8	16.5	m	4.4	28.7	a		25.0	a		25.0	j	7.7	14.8
i	4.5	32.5	i	3.7	29.0	m		12.5	m		12.5	a	7.6	14.8
m	2.7	25.3	j	3.2	14.5	i		49.9	i		49.9	k	6.4	26.0
k	2.2	25.5	k	2.8	27.8	k		37.5	k		37.5	m	5.8	24.0
g	1.2	20.0	g	1.0	17.2	g		25.0	g		24.9	g	4.8	20.4
f	0.6	12.5	f	0.6	13.6	f		0.0	f		0.0	h	1.8	0.9
h	0.5	12.1	h	0.4	9.9	h		12.5	h		12.5	f	1.7	0.0
e	0.2	4.0	e	0.2	4.6	e		0.0	e		0.0	e	1.7	0.0
√ Congest. effect			√ Congest. effect			√ Congest. effect			× Congest effect			× Congest. effect		
√ Evacuation trips			√ Evacuation trips			× Evacuation trips			× Evacuation trips			× Evacuation trips		

†: Ranked by link failure probability in descending order; √: Function available; ×: Function not available

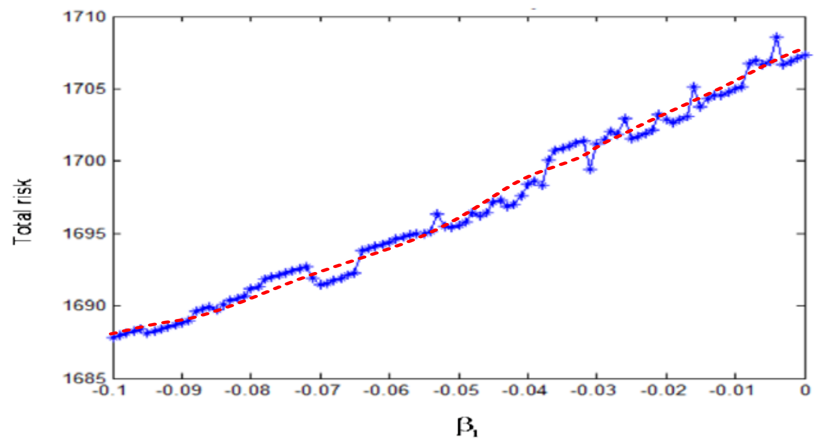
The three models include Qiao's model (2015), Bell's model (2008) and Lownes' model (2011). The results of the three models are compared with our model results in Table 3, where the link ID is listed according to failure probabilities ranked in descending order, and the link use probabilities are also presented. Two sets of results are included for our model based on two levels of risk impact factor β_2. Because the travel demands in the other three models are inelastic, it is assumed that the demand in each OD pair of A-C, A-D, B-C, and B-D is 500 trips for the three models, while in our model only an evacuation demand of 1,000 trips is assumed at origins A and B respectively. At the end of Table 4, the features of each model are described briefly.

As shown in Table 3, in all the four models the most critical two links are link c and link d, which checks well for the effectiveness of our model and its solution strategy. In addition, the following phenomena are observed and are believed to be related to the features of these models. Firstly, in our model, link d is more critical than link c, but the order is opposite in the other models. It might be because that the disturbed link d would lead to more detour cost in our model, and the elastic evacuation demand condition in our model is different from the fixed demand in the other models. Secondly, in Qiao's and Bell's model, except in link c, link d, and link j, other failure probabilities are equal to zero. The reason is that in Qiao's and Bell's models one link is determined to be disturbed rather than using a disruption cost factor and a logit function to reassign the demand for all links in our model. Thirdly, some link use probabilities in Qiao's Bell's, and Lownes' models are equal to zero, however all links in our model are utilized. This may be because of the path/destination choice mechanism in our model that allows the evacuees to choose all possible links and routes to avoid the flooding risk. Fourthly, compared with the model result under a lower risk impact factor (β_2 = -0.5) with a higher impact factor (β_2 = 0), link use probabilities on links with high flooding risks are significantly reduced when evacuees are more sensitive to flooding risk at a higher risk impact factor. For example, the link use probabilities on links *i* and *j* are reduced from 32.5% and 16.8% at β_2 = -0.5 to 29.0% and 14.5% at β_2 = 0, respectively, while the link use probabilities on links *k* and *m* are increased from 25.5% and 25.3% at β_2 = -0.5 to 27.8% and 28.7% at β_2 = 0, respectively. Obviously, the changes of link use probabilities on these links at the two risk impact factors are due to the fact that evacuees are more/less concerned about the higher flooding risks on link *i* and *j* (0.4 and 0.7 respectively) than on links *k* and *m* (0.3 and 0.1 respectively) under the two different risk impact factors.

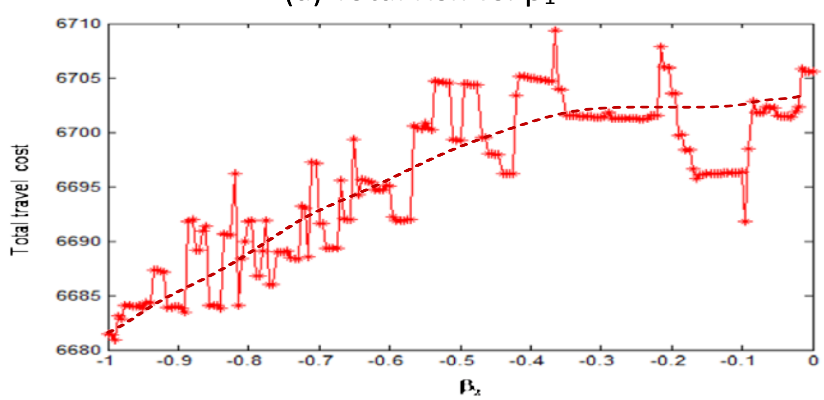
3.3 Impacts on Evacuation Routing

The effects of the consideration on travel cost and flooding risks on evacuation routing are depicted in the total risk vs. β_1 and total cost vs. β_2 curves in Figure 2.

As shown in Figure 2, when the impact factor for travel cost β_1 is increased, the evacuees are more sensitive to travel cost/time spent on the evacuation routes at a higher β_1 than at a lower value. The total risk encountered in all the links of the evacuation paths computed using our model shows an increasing trend along with the increase of the impact factor on travel cost, which means the evacuees are more prone to taking flooding risks in selecting evacuation routes as they are more sensitive to the travel time or cost on the routes.



(a) Total risk vs. β_1



(b) Total cost vs. β_2

Figure 2 Effects of model parameters on network cost and risk

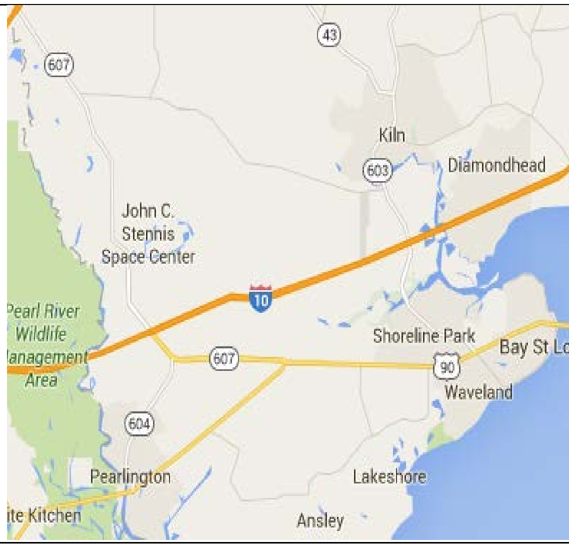
On the other hand, when the impact factor for flooding risk β_2 is increased, the evacuees are more sensitive to flooding risks encountered on the evacuation routes at a higher β_2 value than at a lower value. The total travel time or cost in all the links of the evacuation paths computed using our model shows an increasing trend along with the increase of the impact factor on flooding risk, which means the evacuees are more willing to take detours in selecting less risky evacuation routes as they are more sensitive to the flooding risk on the links and routes.

4 Impacts/Benefits of Implementation

After the test with a sample network, the proposed game model and the solution strategy are applied to a real evacuation network in a case study. The coastal network of Hancock County of the Mississippi Gulf Coast area is used for the case study. The county has a population of 46k most residing near the coast. There are important highway corridors such as I-10, I-59, and US 90 going through or by the study area. The network in the study contains 1,036 links and 439 nodes, and the topological structurer with other information are shown in Figure 3 (a) and (b).

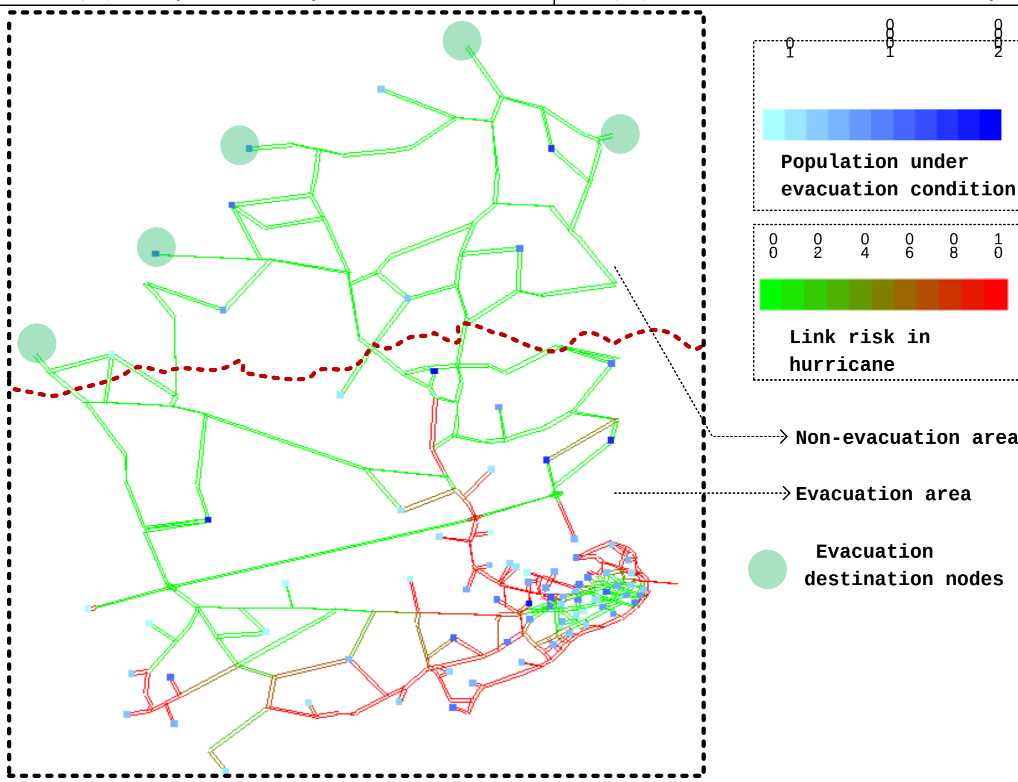
The link flooding risks are calculated by using the Neidoroda method and data (27). The origins of evacuation trip demands are calculated from the social-economic data of Traffic Analysis Zones (TAZ) provided by the Mississippi Department of Transportation (MDOT) and the evacuation destination nodes are determined according to the evacuation routes designated by MDOT or due to the vicinity to a major highway. The population and link risk information of the network in the study area is shown in Figure 3 (c). The emergency scenario is assumed to evacuate the population below the dotted line of the study area in the figure referred to as “evacuation area” to the area above the dotted line referred to as the “non-evacuation area”. Therefore, the traffic trips according to user equilibrium model (UE) in the non-evacuation area are regarded as background traffic for the evacuation operation.

A set of parameters are chosen to set up the inputs for the model, and convergence criterion. The model was coded and run in Matlab 2014 using the same Dell laptop computer as mentioned earlier. The program reaches convergence in a little over 10 minutes.



(a) Map of study area

(b) Road network of study area



(c) Evacuation network in study area

Figure 3 Map and network of study area in Mississippi coast

The computation results for the evacuation network using the proposed game model and solution strategy are shown in Figure 4, where the level of link failure probabilities or critical degrees of links from smallest to largest are illustrated in colors from green to red with red being the most critical. The analysis of the computation results, reveals the following major findings.

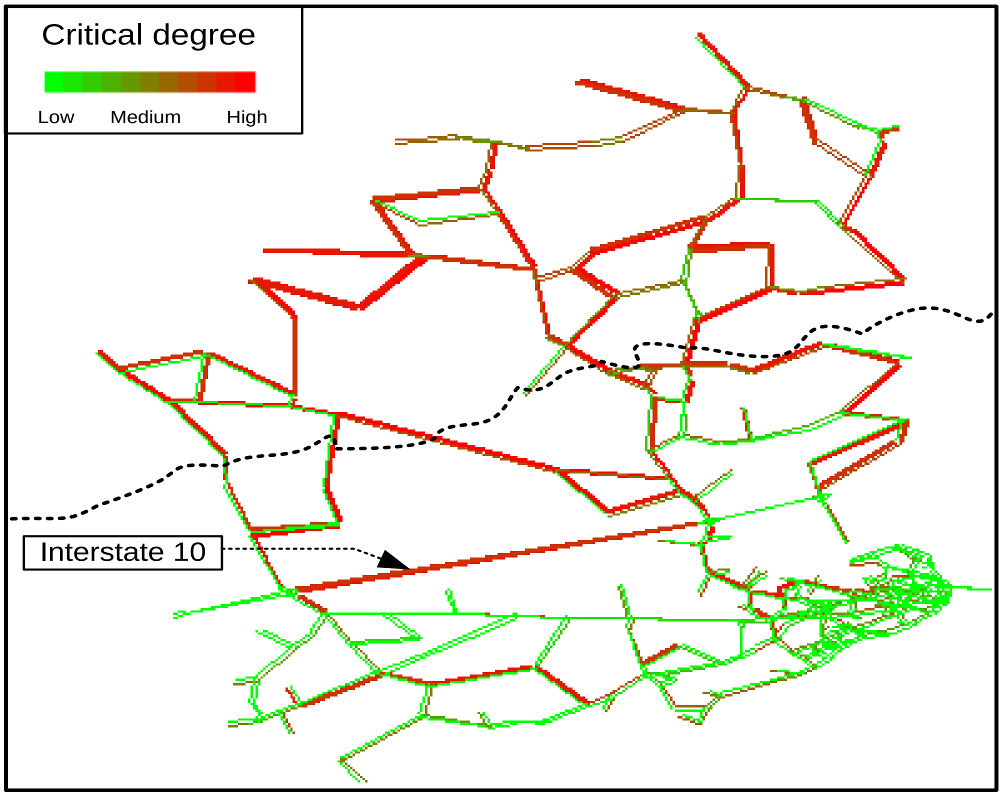


Figure 4 Illustration of critical links of evacuation network

Firstly, in general, the closer the links to the evacuation destination nodes, the higher the link failure probabilities would be, which means these links are more critical than others in the network because the travelers will search for an alternative destination node with more cost induced if the links close to the original destination are disturbed. This finding may suggest that importance and attention be paid to links close to destination nodes to possibly improve evacuation performance. Secondly, the links that direct from the non-evacuation area to the evacuation area are less critical than links in the opposite directions, and the links with high redundancy are less critical than the links with low redundancy. This finding confirms the effectiveness of the traffic control strategies that make use of the less utilized highway capacities. For example the already proved contraflow strategy, which can balance the network flow and improve the throughput efficiency. Thirdly, the failure probabilities of both directions of Interstate 10 are higher than others links inside the evacuation area. This is because of the high capacities of the interstate highway traffic lanes in both directions and any disruption of this corridor would induce much costly detours in rerouting the traffic. Due to the high criticality degree of I-10 in the area, the link should be closely supervised and protected under a hurricane evacuation.

5 Recommendations and Conclusions

Based on the game-theoretic framework, this research study presents an approach to the estimation of vulnerability of a transportation network under hurricane evacuation, especially, when both link risk and evacuation destination choice behavior are considered. To achieve the solution convergence, a heuristic based algorithm using the method of successive averages is developed. In a sample network test, compared with other three models, the model and solution strategy generate reasonable results. The proposed method is applied to the analysis of the vulnerability of an evacuation network in Mississippi coast area under a hurricane invasion, and the link failure probabilities computed using the proposed method can be used to visualize the degree of link criticality for the evacuation scenario and the link flooding risks of the network.

The total risk encountered in all the links of the evacuation paths computed using the proposed model shows an increasing trend along with the increase of the impact factor on travel cost, which means the evacuees are more prone to taking flooding risks in selecting evacuation routes as they are more sensitive to the travel time or cost on the routes. On the other hand, the total travel time or cost in all the links of the evacuation paths shows an increasing trend along with the increase of the impact factor on flooding risk, which means the evacuees are more willing to take detours in selecting less risky evacuation routes as they are more sensitive to the flooding risk on the links and routes. The analysis of the evacuation network in Mississippi coast area using the proposed method suggests that links near the evacuation destinations tend to be more critical, and important traffic corridors such as I-10 in the evacuation network has a high degree of criticality.

There are two challenges for the study in the future. Firstly, the risk/cost impact factors may not be the same for different areas or evacuees, and need more data and research to evaluate and understand these factors. Secondly, although the evacuation demand is elastic, the time dependent effect is not considered in this model. If the time dependent effect should be included, a dynamic evacuation behavior in route choice would be represented.

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